

2024

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION



Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11–32 show relevant mathematical reasoning and/or calculations
- Write your NESA ID below, on the Multiple Choice Answer Sheet and the front of Booklets 1, 2, 3 & 4.

Total marks:
100

Section I – 10 marks (pages 3-9)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 10-38)

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

NESA NUMBER:

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TEACHER: _____

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Multiple Choice:	/3 3, 7, 10	/1 1	/3 4, 5, 9		/2 6, 8	/1 2

Multiple Choice & Questions 11-32	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics	TOTAL
	/29	/12	/23	/5	/16	/15	/100

Section I

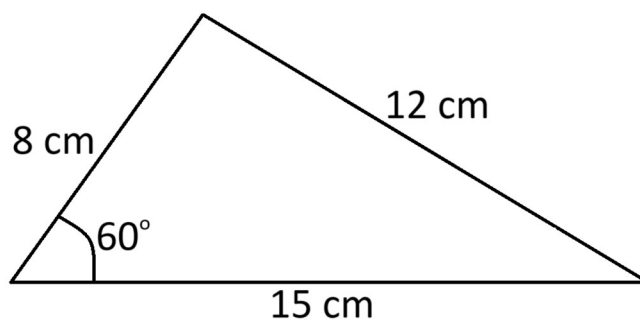
10 marks

Attempt questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

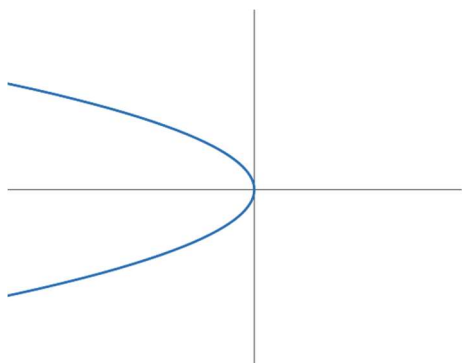
- 1 The area of the triangle shown below is:



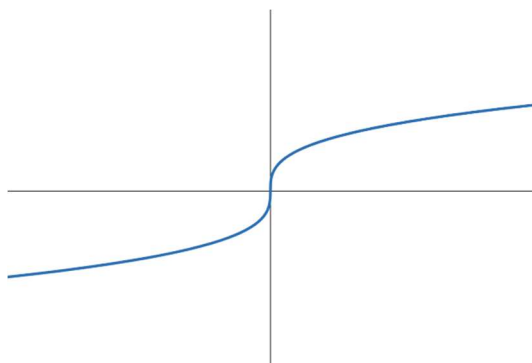
- A. 30 cm^2
- B. $24\sqrt{3} \text{ cm}^2$
- C. $30\sqrt{3} \text{ cm}^2$
- D. 52 cm^2
- 2 Find the 30th term in the sequence $-8, -2, 4 \dots$
- A. 166
- B. 172
- C. 1392
- D. 2370

3 Which of the following is a graph of a one-to-many function?

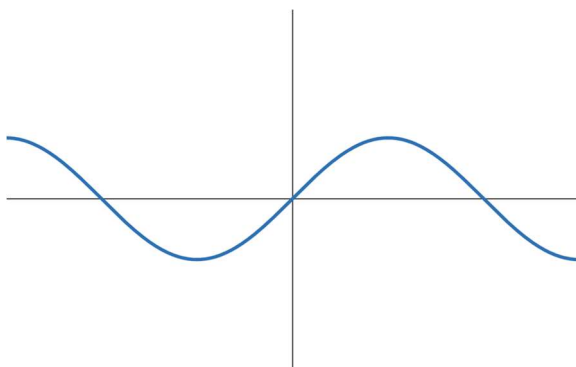
A.



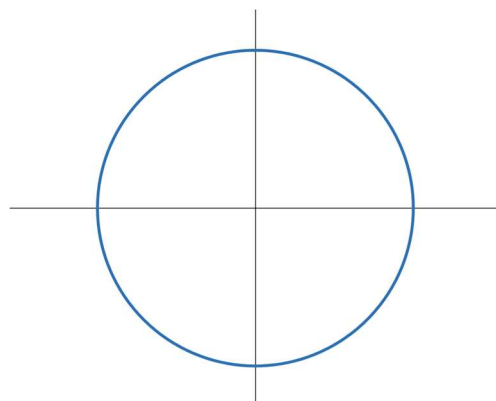
B.



C.



D.



4 Find $\int 3^{(6x+5)} dx$

A. $\frac{3^{(6x+5)}}{\ln 3} + C$

B. $\frac{3^{(6x+5)}}{6 \ln 3} + C$

C. $\frac{3^{(6x+5)}}{\ln 18} + C$

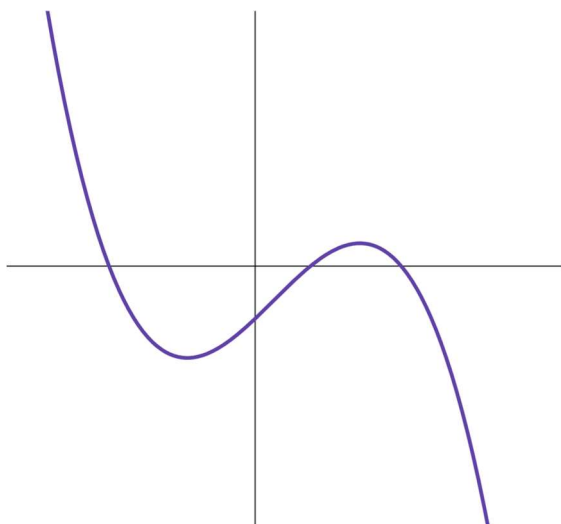
D. $\frac{18^{(6x+5)}}{\ln 3} + C$

- 5 The graph of a function $y = f(x)$ is shown below.

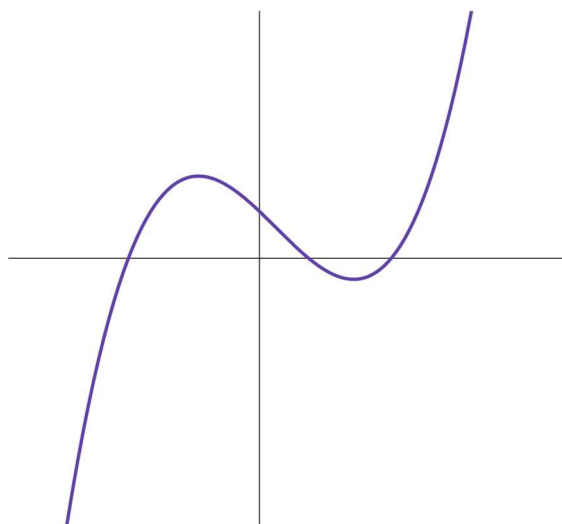


Which of the following is the graph of $y = f'(x)$?

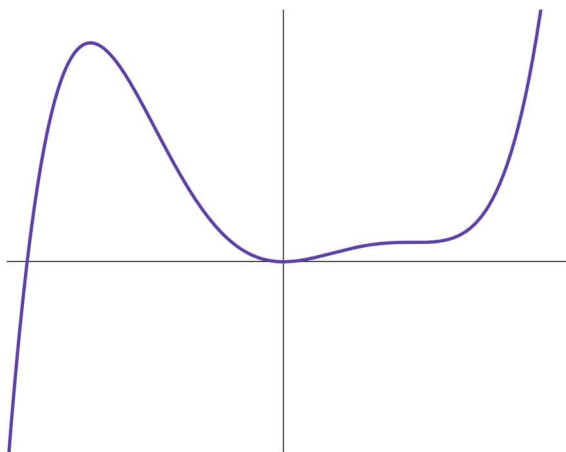
A.



B.



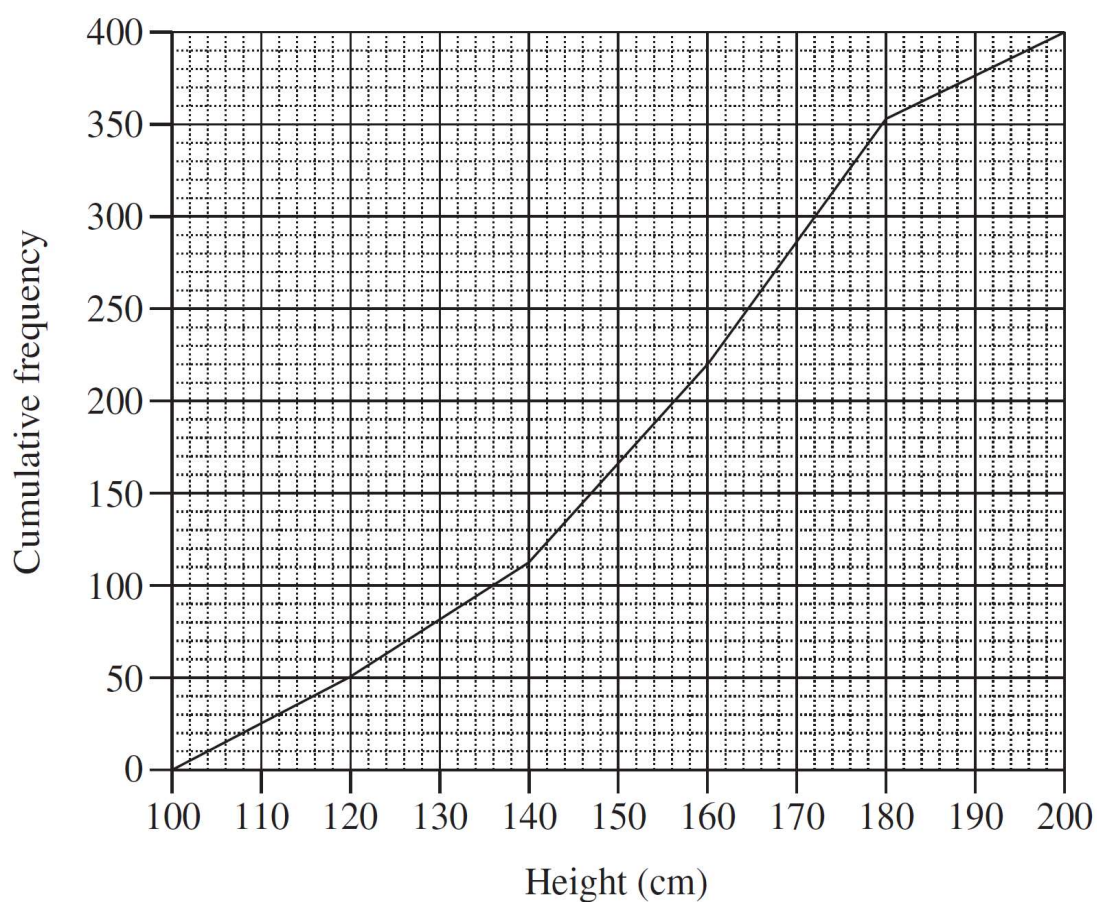
C.



D.



- 6 The height of 400 students were measured. The results are displayed in the cumulative frequency polygon shown below.

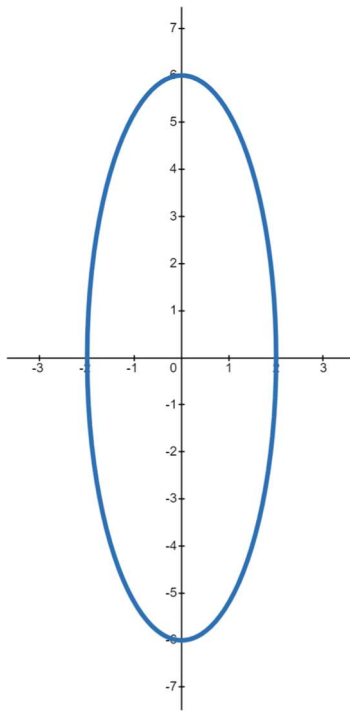


The interquartile range of the height of the students is.

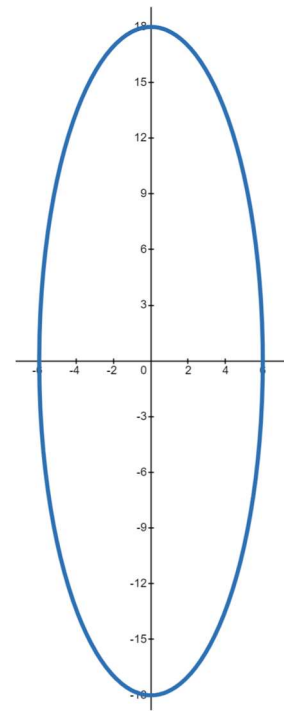
- A. 36
- B. 38
- C. 100
- D. 200

7 Which of the following best represents the graph of $(3x)^2 + y^2 = 36$?

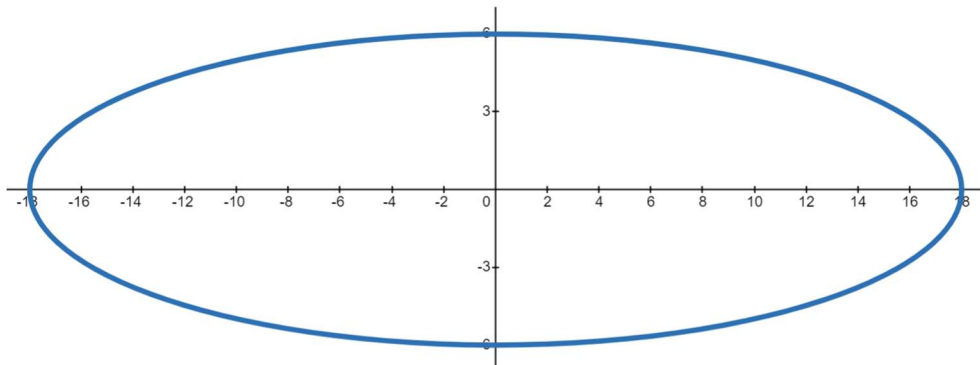
A.



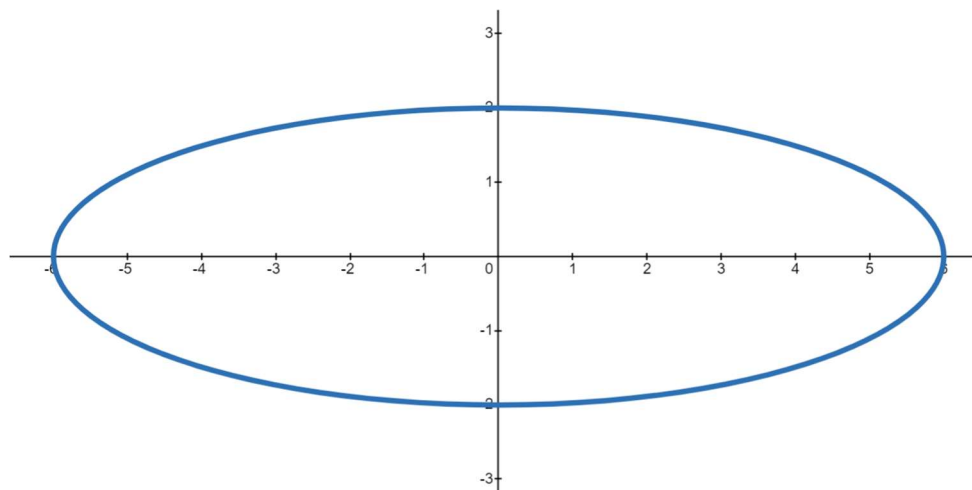
B.



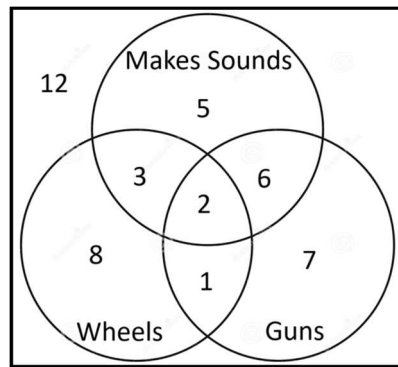
C.



D.

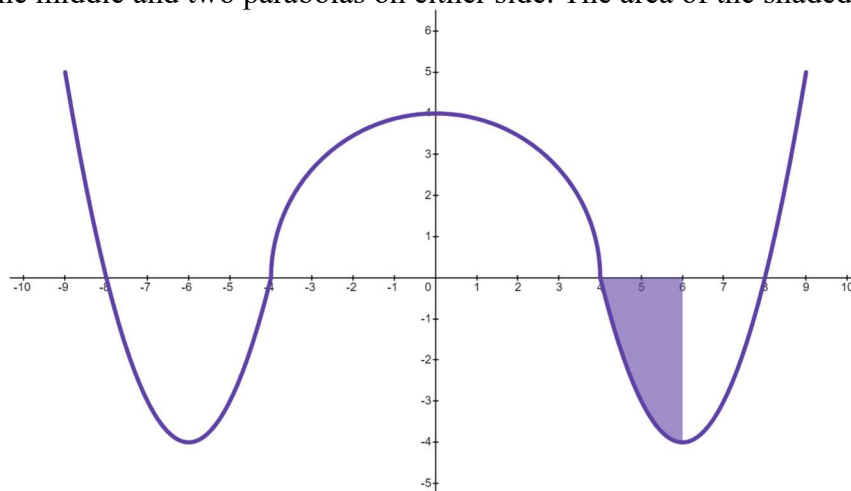


- 8 Jim sorted out his toys by their features, as shown in the Venn Diagram below.



Given that Jim selects a toy with wheels, find the probability that it also makes sounds.

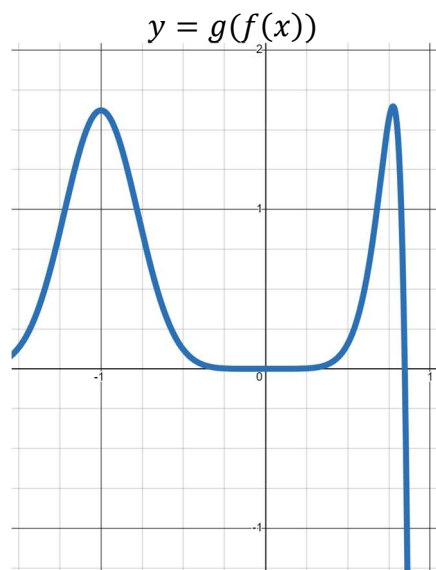
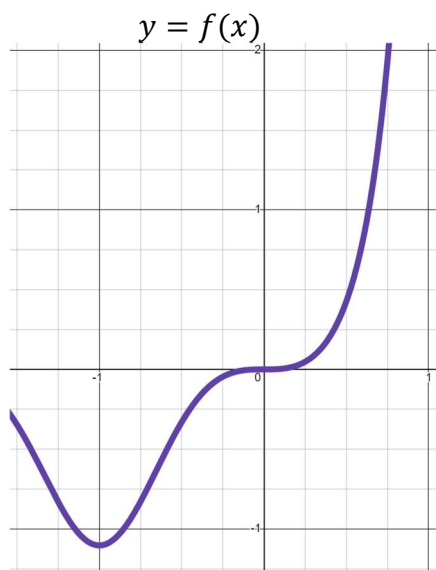
- A. $\frac{3}{8}$
 B. $\frac{3}{14}$
 C. $\frac{5}{14}$
 D. $\frac{5}{44}$
- 9 The diagram shows the graph of an even function $y = f(x)$. The function is made up of a semicircle in the middle and two parabolas on either side. The area of the shaded region is 2.



What is the value of $\int_{-8}^8 f(x) dx$ correct to 1 decimal place?

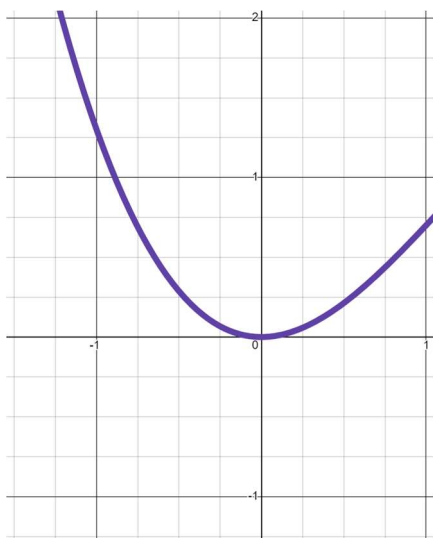
- A. 17.1
 B. 20.6
 C. 33.1
 D. 36.3

- 10 The graphs of $y = f(x)$ and $y = g(f(x))$ are shown.

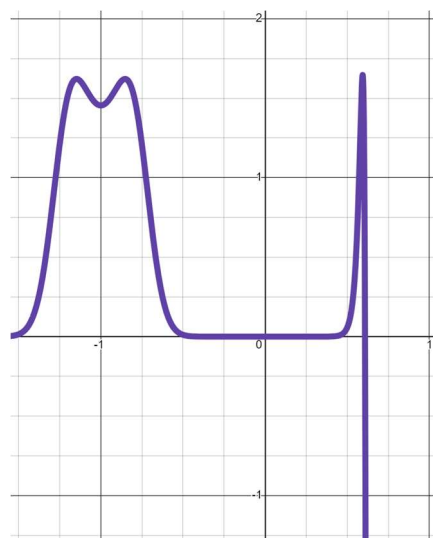


Which graph best represents $y = g(x)$?

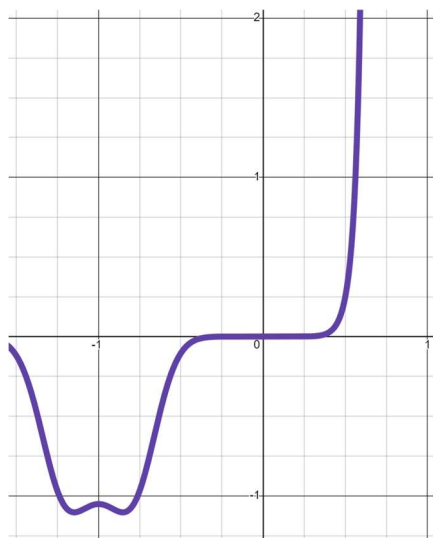
A.



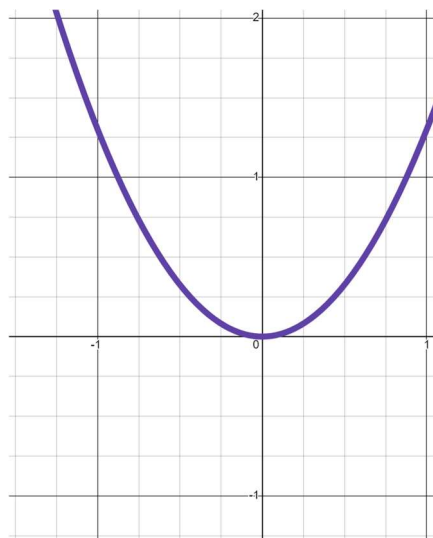
B.



C.



D.



Question 11 (2 marks)

Solve $\frac{3x+2}{5} - \frac{1-2x}{3} = 8$.

2

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 12 (2 marks)

Given that $\frac{12 + \sqrt{32}}{3\sqrt{2}} = a + b\sqrt{2}$. Show that $a = \frac{4}{3}$ and $b = 2$.

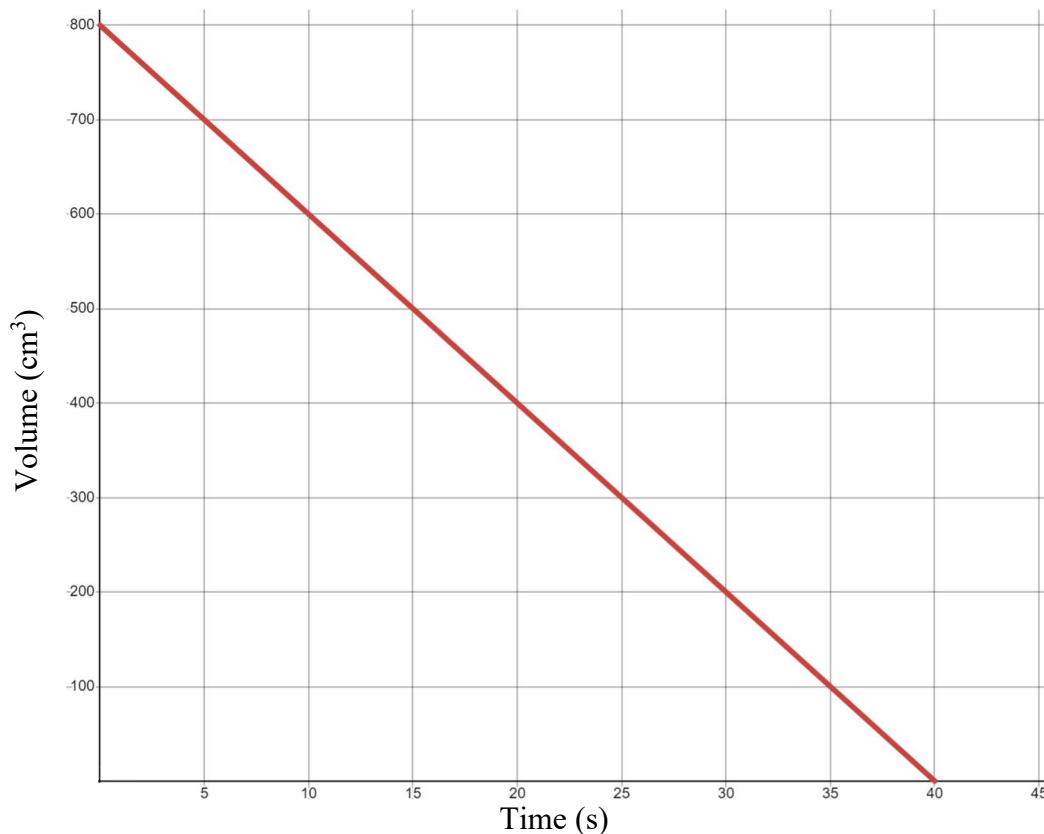
2

[illegible]

Question 13 (3 marks)

Owais has two balloons, Balloon A and Balloon B. Initially Balloon A is filled with a volume, V , of 800 cm^3 of air and Balloon B is empty.

Owais accidentally puts a hole in Balloon A, which causes it to deflate over t seconds, as shown in the graph below.



- (a) At what rate is Balloon A deflating?

1

- (b) After 10 seconds of Balloon A deflating, Owais' mum starts to inflate Balloon B. She does this at a rate of $40 \text{ cm}^3/\text{s}$.

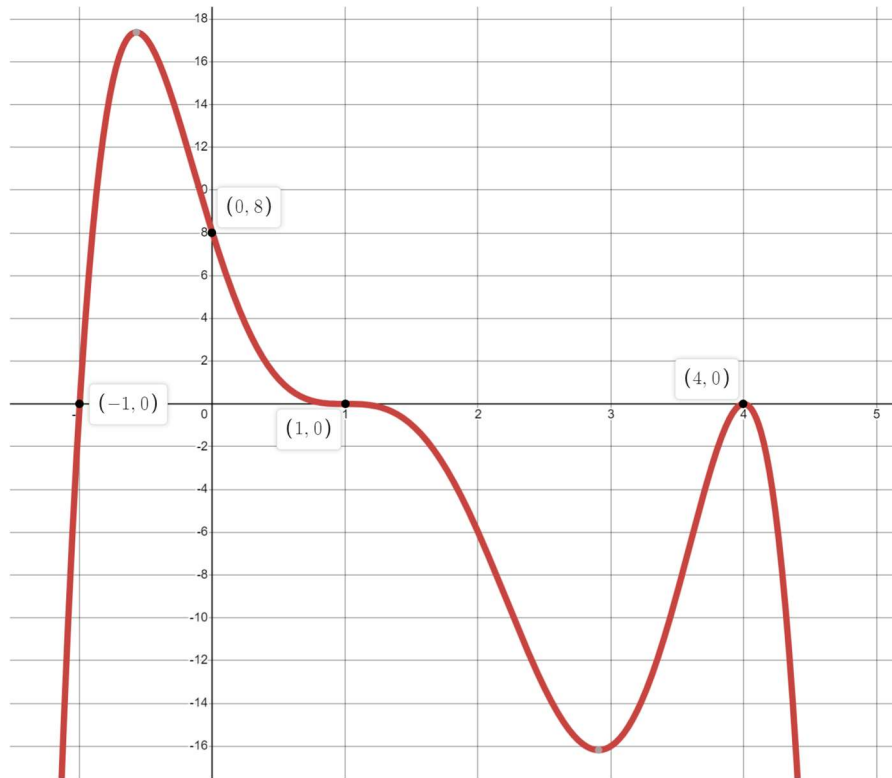
2

By drawing a line on the grid (above), find the value of t when the two balloons contain the same amount of air.

Question 14 (3 marks)

Find the equation of the polynomial with degree 6.

3

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

Question 15 (4 marks)

The probability that Darcy wants to play with a football outside on any particular day is $P(F)$. The probability that it rains on any particular day is $P(R)$.

It is given that $P(F) = \frac{2}{3}$, $P(F \cap R') = \frac{1}{2}$ and $P(F' \cap R') = \frac{1}{4}$.

- (a) Is Darcy wanting to play with a football outside independent from whether or not it is raining? Justify your answer.

2

[illegible]

- (b) Given that these probabilities stay the same every day, how many days would it take to ensure a 98% chance of Darcy wanting to play football at least once?

2

[illegible]

Question 16 (3 marks)

(a) Find $\frac{d}{dx}(x^3 + 1)^4$.

2

(b) Hence, find $\int 4x^2(x^3 + 1)^3 dx$.

1

Question 17 (5 marks)

On any given night, the number X of times that Isaac wakes up is a random variable with probability distribution given by:

x	0	1	2	3	4
$P(X = x)$	0.05	0.3	0.42	0.2	0.03

- (a) Show that Isaac's parents should expect him to wake up 1.86 times a night.

1

- (b) Calculate the standard deviation, correct to one decimal place.

2

- (c) Find the probability that Isaac wakes up greater than the expected number of times for three days in a row.

2

Section II extra writing space

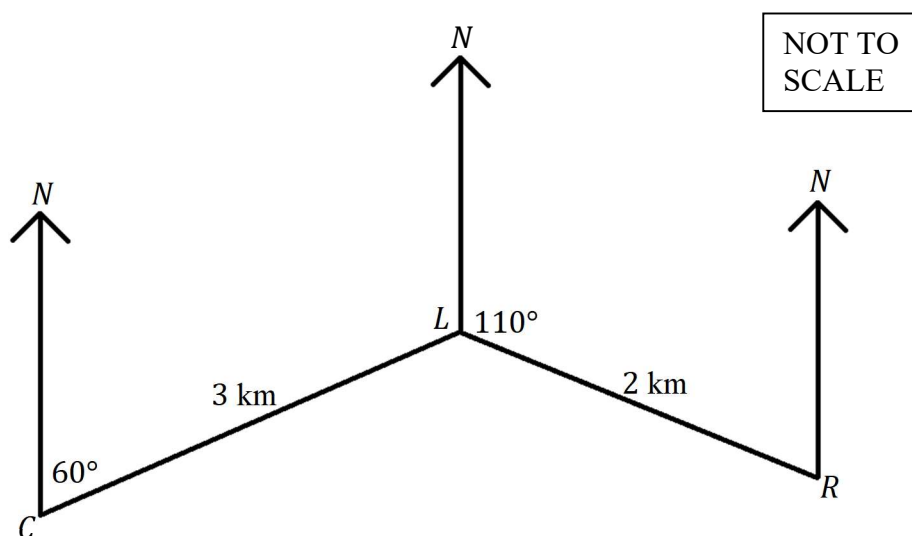
If you use this space, clearly indicate which question you are answering.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 18 (4 marks)

Henry is going on a bushwalk. He walks 3 km on a bearing of $060^\circ T$ from his car, C , where he reaches a lookout, L . He then walks 2 km on a bearing of 110° to a rockpool, R .

4



Find the shortest distance that Henry needs to walk to return to his car and the bearing that he would be walking on from the Rockpool (to the nearest degree).

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Question 19 (4 marks)

Elena buys a car for \$25 000 and repays it over 4 years through equal monthly instalments. She is charged interest at 7.8% p.a. compounded monthly on the reducing balance loan.

Using the table of present value interest factors below, where r represents the monthly interest and N represents the number of repayments:

Table of Present Value Interest Factors						
r	0.0060	0.0065	0.0070	0.0075	0.0080	0.0085
N						
45	39.33406	38.90738	38.48712	38.07318	37.66545	37.26383
46	40.09350	39.64965	39.21263	38.78231	38.35859	37.94133
47	40.84841	40.38714	39.93310	39.48617	39.04622	38.61311
48	41.59882	41.11986	40.64856	40.18478	39.72839	39.27924
49	42.34475	41.84785	41.35905	40.87820	40.40515	39.93975
50	43.08623	42.57113	42.06459	41.56645	41.07653	40.59470

- (a) Calculate the monthly repayment that Elena must pay to complete the loan after 4 years. 2

- (b) Calculate the total interest paid over the life of the loan. 2

Question 20 (3 marks)

Find the equation of the tangent to $y = \frac{x^4}{x-6}$ at the point $(2, -4)$.

3

Question 21 (3 marks)

For which values of k does $y = 2x^2 + kx + 8$ have real roots?

3

Question 22 (6 marks)

Aimee was baking some biscuits. She decided to make different sized biscuits, B , in grams and compare the time taken to bake them, t , in minutes. Aimee recorded her results in a table of values.

She then calculated the least squares regression line to be $t = 1.23B - 7.13$ and Pearsons correlation coefficient to be $r = 0.992$.

However, after doing these calculations she accidentally spilled her coffee on her notes and now can not see the row where she recorded the biscuit sizes. Conveniently she did not spill any coffee on the row where she recorded the time take to cook. Her table of values (with coffee stain) is shown below.

Biscuit Size (B)						
Time to cook (t)	12	16	17	20	22	25

Aimee knows that in any bivariate dataset, the least squares regression line passes through the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the sample mean of the x -values and $\bar{y}$ is the sample mean of the y values.

- (a) Use this information to find the average weight of the biscuits.

2

- (b) Describe and interpret the gradient of the least squares regression line with reference to the context given.

1

Question 22 continues on the next page.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

NESA Number:

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2024

**Penrith High School
HSC TRIAL
EXAMINATION**

Teacher: _____

Mathematics Advanced Section II Answer Booklet 3

Booklet 3 – Attempt Questions 24 – 29 (25 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

Booklet 3	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
	/13 24, 28, 29	/7 27		/5 25, 26		

Please turn over

Question 24 (5 marks)

The graph of the function $f(x) = x^3$ is translated up 9 units, left 4 units, then dilated vertically by a scale factor of $\frac{1}{3}$, to make a new graph, $g(x)$.

- (a) Find the equation, $g(x)$, of the new graph.

3

- (b) A point $P(a, b)$ on $f(x)$ is transformed to the point $Q(-1, 12)$ on $g(x)$.
Find the values of a and b .

2

Question 25 (2 marks)

The difference in sound levels in decibels (dB) can be modelled by the equation

2

$$D_2 - D_1 = 10 \log_{10} \left(\frac{I_2}{I_1} \right)$$

Where D_1 and D_2 are the sound levels and I_1 and I_2 are their respective sound intensities measured in watts/m².

A conversation has a sound level of 60 dB. A vacuum cleaner has an intensity that is 100 times more than the conversation. Calculate the sound level of the vacuum cleaner.

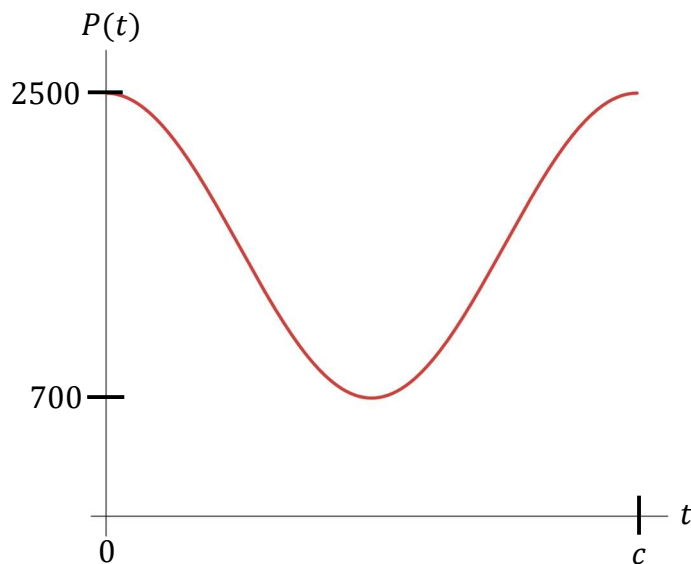
Question 26 (3 marks)

Given $2 \log_4 y - \log_4 8 + \log_4 x = 2$, show that $y = \frac{8\sqrt{2}}{\sqrt{x}}$.

3

Question 27 (6 marks)

Anubhav is doing some research on the number of patients emergency waiting rooms across all NSW hospitals at any given time. He notices that the number of patients, P , can be modelled by the function $M(t) = a \cos\left(\frac{\pi t}{6}\right) + b$, where t is the time since 12 am. The maximum number of patients is 2500 and the minimum is 700. The graph of $P(t)$ is shown below.



- (a) What are the values of a and b in the equation and c on the graph?

3

- (b) Between what times (to the nearest minute) each day are there less than 1000 patients in emergency waiting rooms across all NSW hospitals?

4

Question 28 (5 marks)

Carlisle is playing a game, in which he has to position a bomb anywhere on a number plane. When the bomb explodes it will kill anything within its circular blast zone.

- (a) Find the position that Carlisle placed the bomb on the number plane and the radius of its circular blast zone, given its' blast zone has an equation $x^2 - 20x + y^2 + 10y = -76$.

3

- (b) Determine if an enemy located at the coordinates $(2, -4)$ will be killed when the bomb explodes. Justify your decision with a diagram and/or calculations.

2

Question 29 (3 marks)

Let $f(x) = \sqrt{3x - 15}$ and $g(x) = -6x - 4$.

(a) Find $h(x)$, where $h(x) = f(g(x))$.

1

(b) State the domain for $h(x)$.

2

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

NESA Number:

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2024

**Penrith High School
HSC TRIAL
EXAMINATION**

Teacher: _____

Mathematics Advanced Section II Answer Booklet 4

Booklet 3 – Attempt Questions 30 – 32 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 4			32 /12			30, 31 /10

Please turn over

Question 30 (2 marks)

Jacinta is gifted \$5000 from her grandmother. The money is invested for 18 years, earning 7% p.a. in interest, compounded monthly. Calculate the effective annual interest rate of the investment over the 18 years. Answer as a percentage correct to two decimal places.

2

Question 31 (8 marks)

Ashley borrows \$500 000 from a bank in a reducing balance loan for which she is charged an interest rate of 6% per annum, compounded monthly. At the end of each month Ashley is required to make a repayment of \$4000.

The amount in the account immediately after the n th repayment can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - 4000$$

where $n = 1, 2, 3 \dots$ and $A_0 = 500\,000$.

- (a) Use the recurrence relation to determine the amount of money owing immediately after the third repayment.

2

Question 31 continues on the next page.

(c) Hence, find how many months it will take for Ashley to repay the \$500 000.

3

Question 32 (13 marks)

Let $f(x) = 3x^3 e^x$.

(a) Show that $f'(x) = 3e^x (3x^2 + x^3)$.

2

Question 32 continues on the next page.

3

- (b) Find any points of inflection of $f(x)$.
(You may state the values of the points correct to 1 decimal place.)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 32 continues on the next page.

- (c) Show that the only stationary points of $f(x)$ are $(0, 0)$ and $(-3, -81e^{-3})$ and determine their nature.

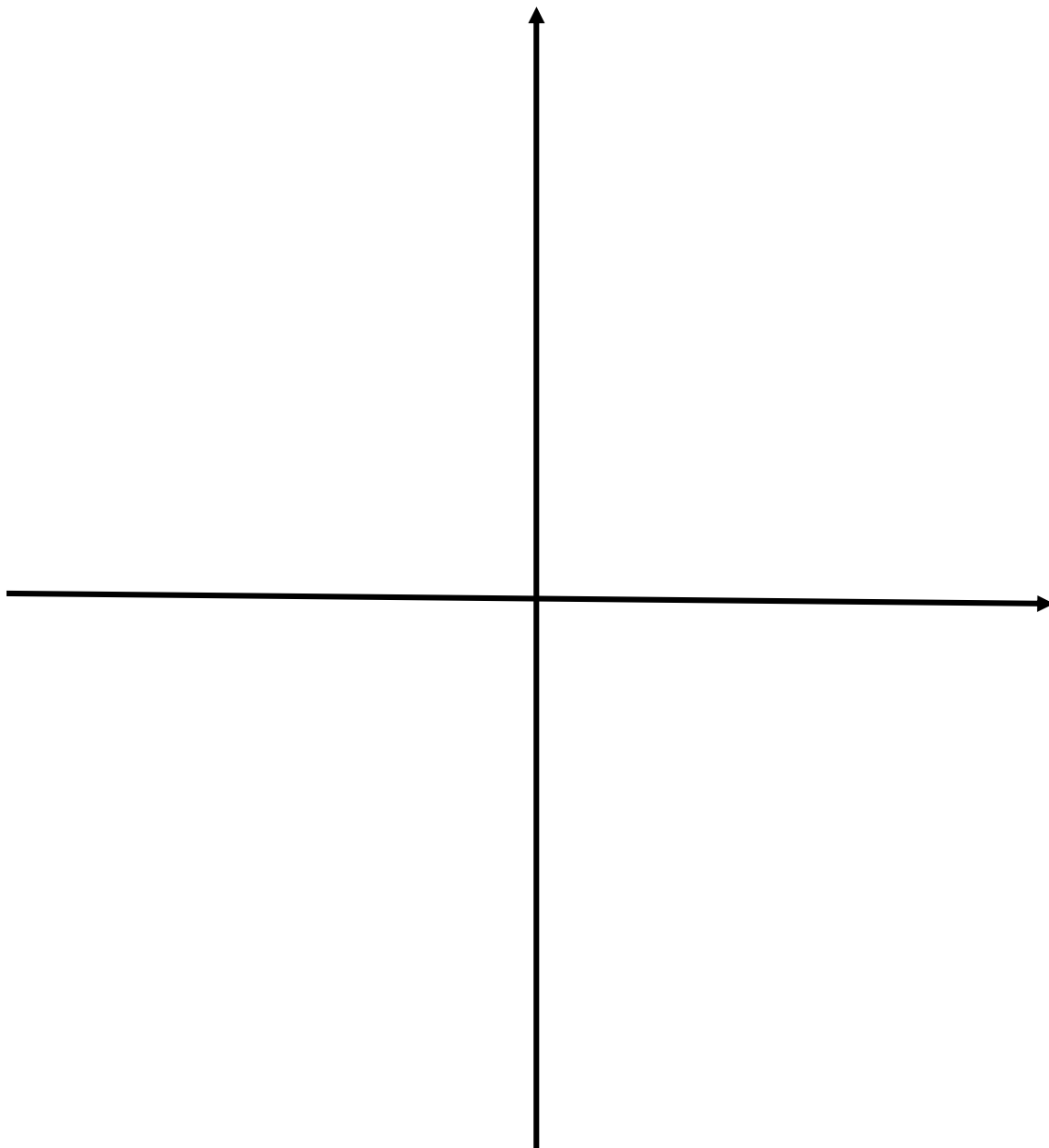
4

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Question 32 continues on the next page.

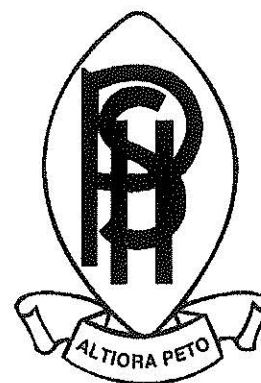
- (d) Hence sketch the curve of $f(x) = 3x^3 e^x$ for $x < 1$, showing any stationary points, points of inflection and intercepts with the axes.

3



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Mathematics Advanced

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TEACHER: _____

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Multiple Choice:	/3 3, 7, 10	/1 1	/3 4, 5, 9		/1 6, 8	/1 2

Multiple Choice & Questions 11-32	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics	TOTAL
	/29	/11	/23	/5	/20	/11	/100

Section I

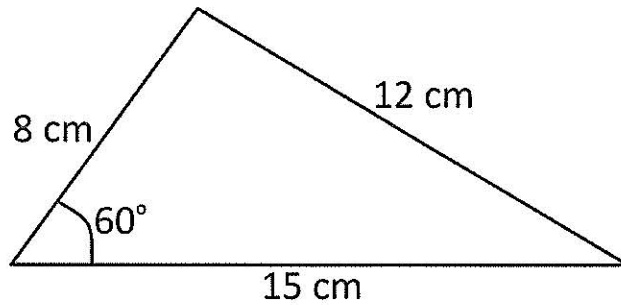
10 marks

Attempt questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

- 1 The area of the triangle shown below is:



- A. 30 cm^2
B. $24\sqrt{3} \text{ cm}^2$
☒ C. $30\sqrt{3} \text{ cm}^2$
D. 52 cm^2

$$\begin{aligned} A &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}(8)(15)\sin 60 \\ &= 60 \times \frac{\sqrt{3}}{2} \\ &= 30\sqrt{3} \end{aligned}$$

- 2 Find the 30th term in the sequence $-8, -2, 4 \dots$

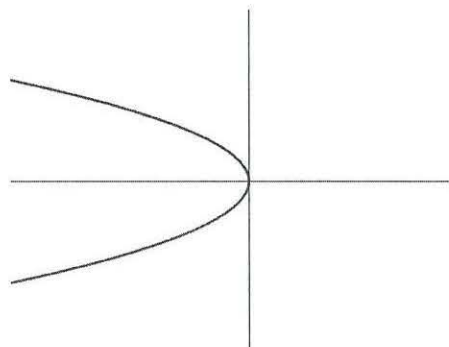
- ☒ A. 166
B. 172
C. 1392
D. 2370

$$a = -8, d = 6$$

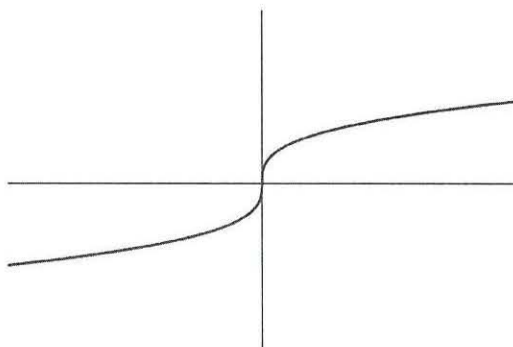
$$\begin{aligned} T_{30} &= -8 + (30-1) \times 6 \\ &= 166 \end{aligned}$$

3 Which of the following is a graph of a one-to-many function?

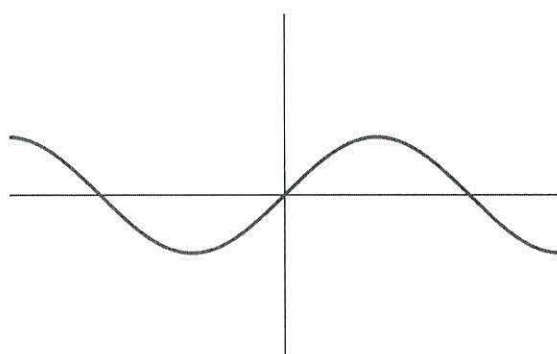
A.



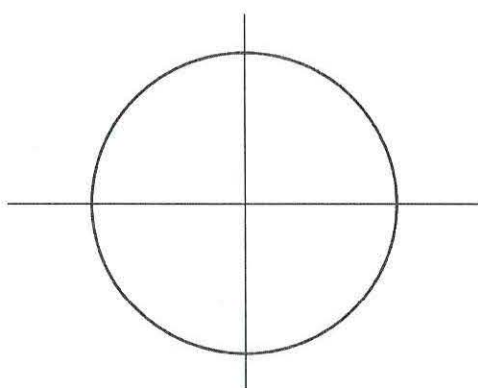
B.



C.



D.



4 Find $\int 3^{(6x+5)} dx$

A. $\frac{3^{(6x+5)}}{\ln 3} + C$

B. $\frac{3^{(6x+5)}}{6 \ln 3} + C$

C. $\frac{2^{(6x+5)}}{\ln 3} + C$

D. $\frac{18^{(6x+5)}}{\ln 3} + C$

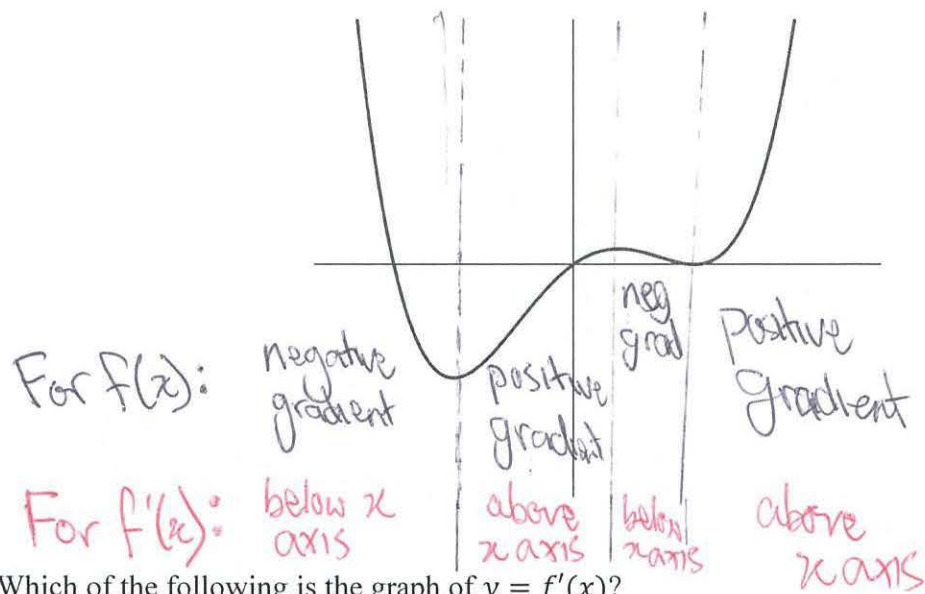
$$f(x) = 6x + 5$$

$$f'(x) = 6$$

$$\frac{1}{6} \int 6 \times 3^{6x+5} dx$$

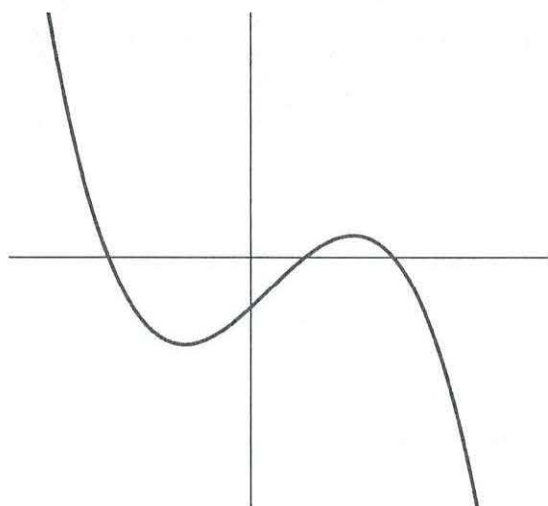
$$= \frac{1}{6} \times \frac{3^{6x+5}}{\ln 3} + C$$

- 5 The graph of a function $y = f(x)$ is shown below.

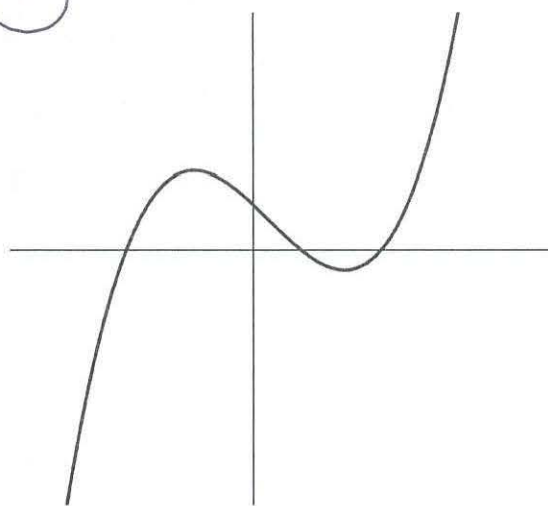


Which of the following is the graph of $y = f'(x)$?

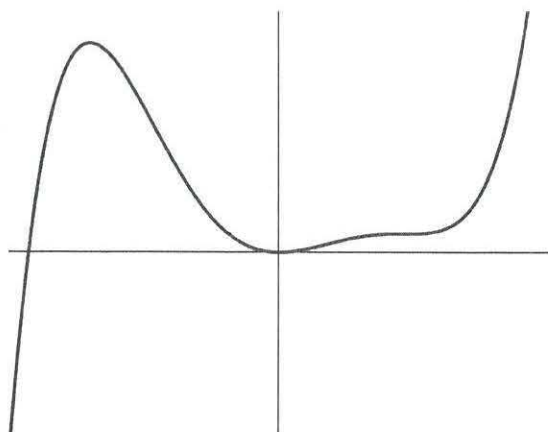
A.



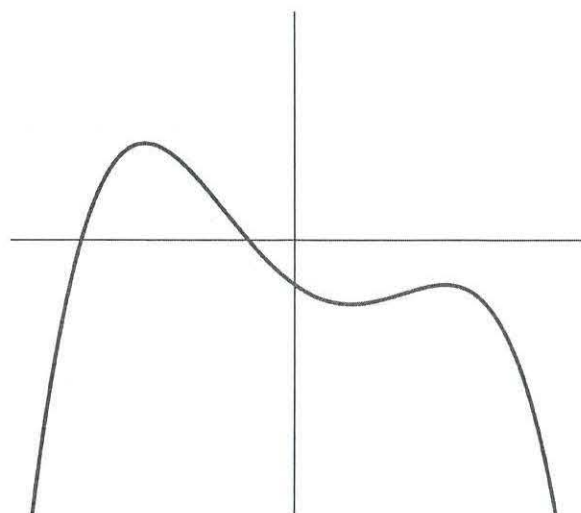
B.



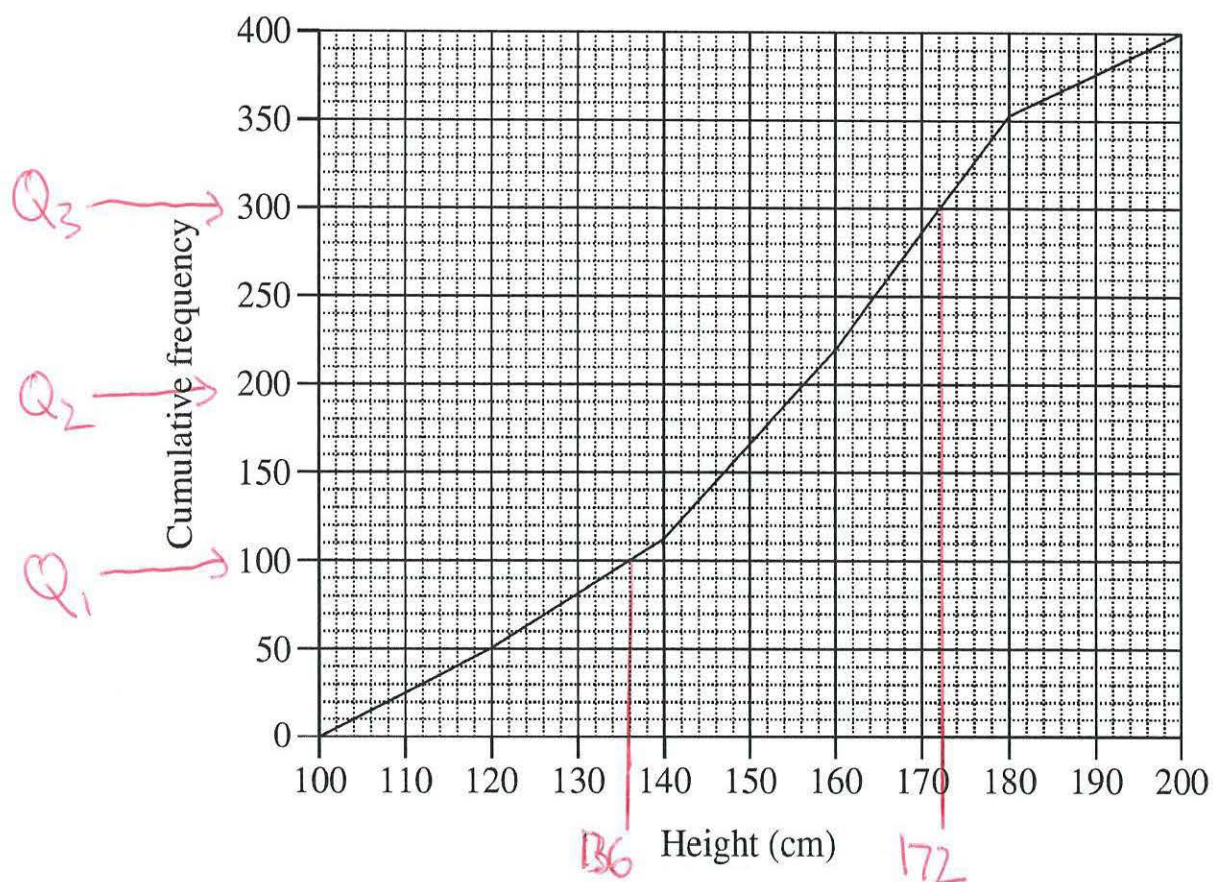
C.



D.



- 6 The height of 400 students were measured. The results are displayed in the cumulative frequency polygon shown below.



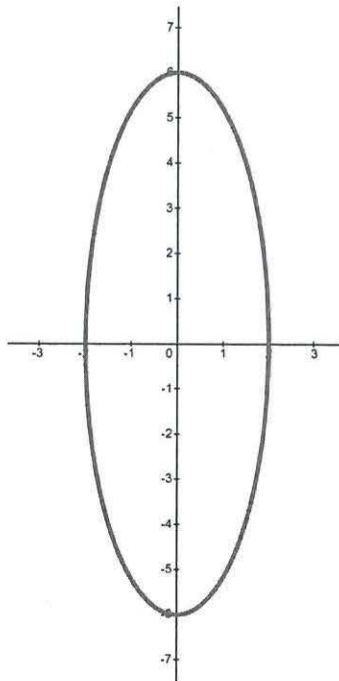
The interquartile range of the height of the students is.

- A. 36
- B. 38
- C. 100
- D. 200

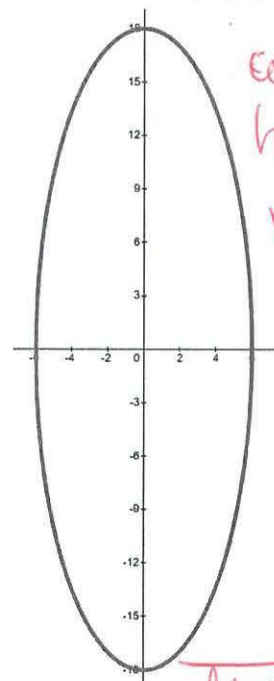
$$\begin{aligned}
 IQR &= Q_3 - Q_1 \\
 &= 172 - 136 \\
 &= 36
 \end{aligned}$$

7 Which of the following best represents the graph of $(3x)^2 + y^2 = 36$?

A.



B.



Horizontal dilation by a factor of $\frac{1}{3}$.

∴ Circle is

compressed horizontally, making the left/right extremes at ± 2 now instead of ± 6 .

ALTERNATE METHOD:

- x-int when $y=0$

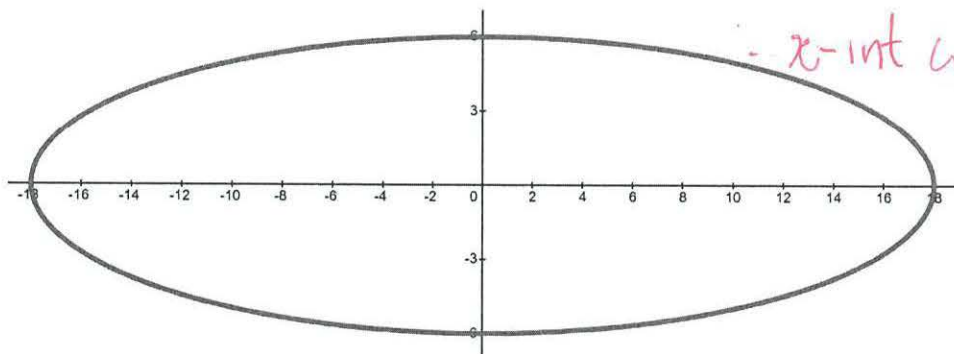
$$(3x)^2 + 0^2 = 36$$

$$9x^2 = 36$$

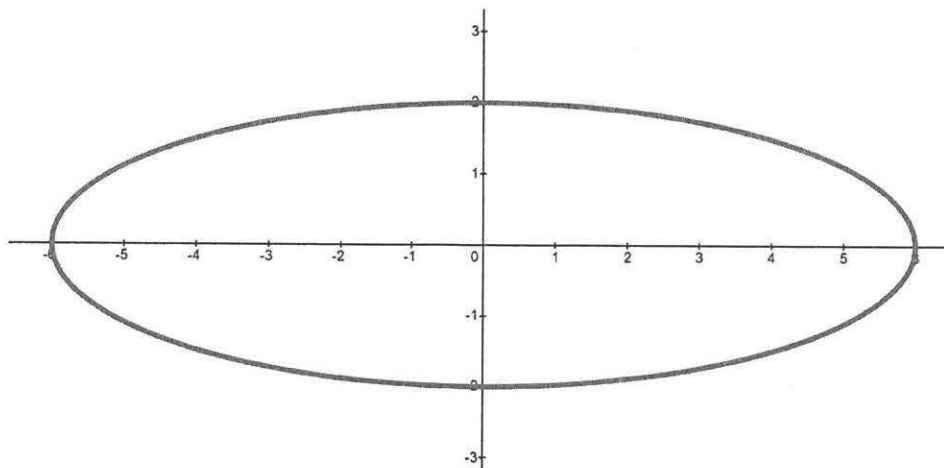
$$x^2 = 4$$

$$x = \pm 2$$

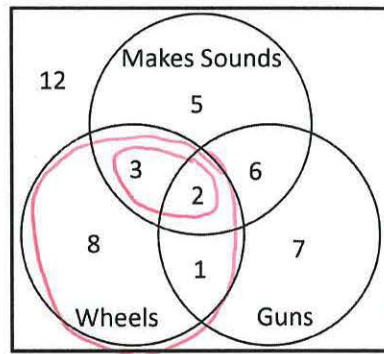
C.



D.



- 8 Jim sorted out his toys by their features, as shown in the Venn Diagram below.

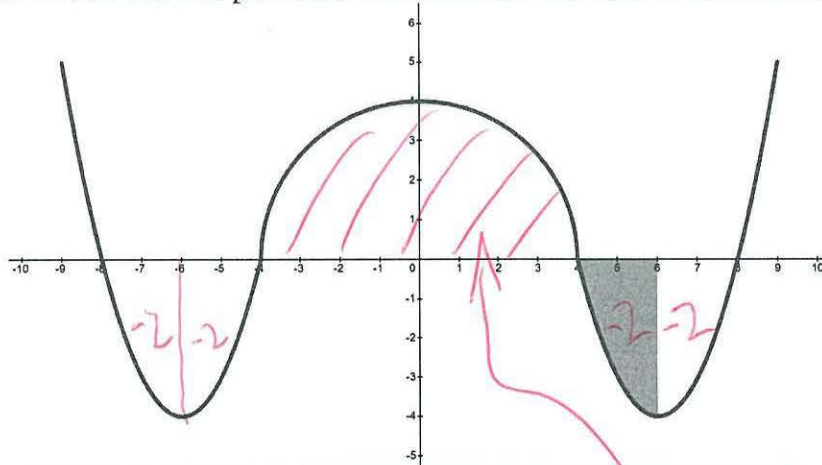


Given that Jim selects a toy with wheels, find the probability that it also makes sounds.

- A. $\frac{3}{8}$
 B. $\frac{3}{14}$
 C. $\frac{5}{14}$
 D. $\frac{5}{44}$

$$\frac{3+2}{3+2+8+1}$$

- 9 The diagram shows the graph of an even function $y = f(x)$. The function is made up of a semicircle in the middle and two parabolas on either side. The area of the shaded region is 2.



What is the value of $\int_{-8}^8 f(x) dx$ correct to 1 decimal place?

- A. 17.1
 B. 20.6
 C. 33.1
 D. 36.3

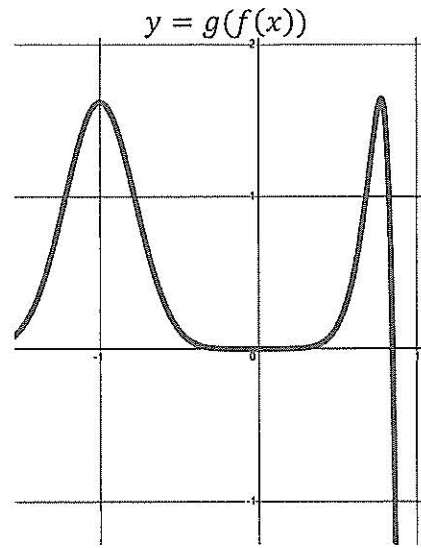
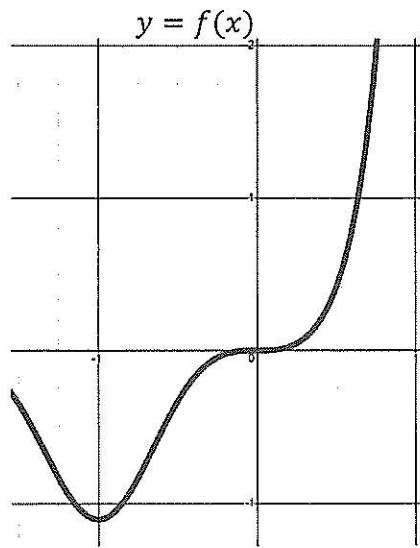
$$= 8\pi + (-2 \times 4)$$

$$= 17.13$$

$$A = \frac{1}{2} \pi (4)^2$$

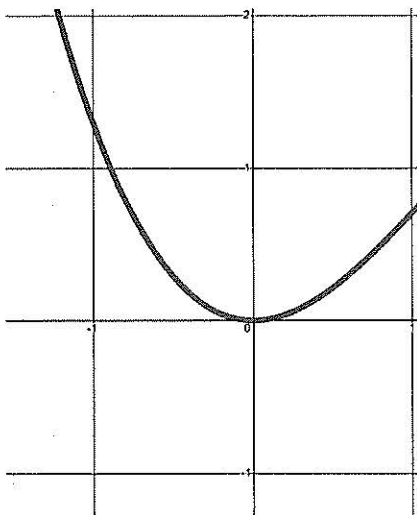
$$= 8\pi$$

- 10 The graphs of $y = f(x)$ and $y = g(f(x))$ are shown.

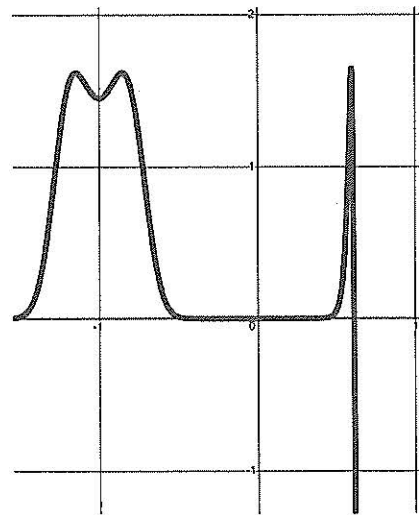


Which graph best represents $y = g(x)$?

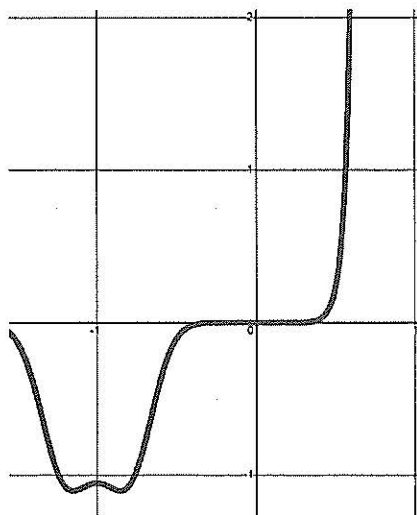
A.



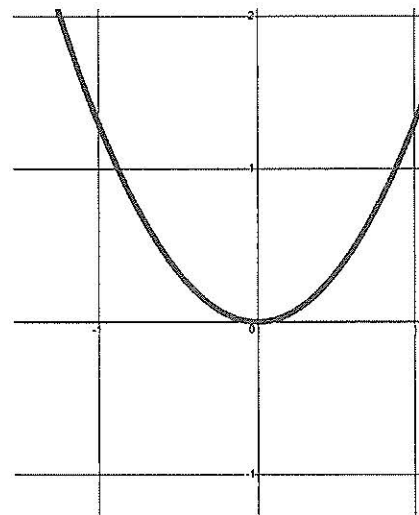
B.



C.



D.



2024 Trial HSC Q10 Working Out

$y = f(x)$ means substitute x -values into $f(x)$ to get y -values.

$y = g(f(x))$ means substitute x values into $f(x)$, then substitute these $f(x)$ values into $g(x)$, to get the final y -values.

Now let's test a point, say when $x = 0.5$.

$$x = 0.5 \rightarrow f(x) \approx 0.4 \text{ and } g(f(x)) \approx 0.2$$

So $y = g(x)$ should include the point $(0.4, 0.2)$.

(As stated above, for $y = g(f(x))$, substitute the $f(x)$ values into $g(x)$)

Option A: $x = 0.4 \rightarrow g(x) \approx 0.2$

Option B: $x = 0.4 \rightarrow g(x) \approx 0.001$ (Eliminate this option)

Option C: $x = 0.4 \rightarrow g(x) \approx 0.2$

Option D: $x = 0.4 \rightarrow g(x) \approx 0.25$ (Eliminate this option)

We now have Option A and C left.

Let's test another point. say when $x = -0.5$.

$$x = 0.5 \rightarrow f(x) \approx -0.3 \text{ and } g(f(x)) \approx 0.175$$

So $y = g(x)$ should include the point $(-0.3, 0.175)$.

(As stated above, for $y = g(f(x))$, substitute the $f(x)$ values into $g(x)$)

Option A: $x = -0.3 \rightarrow g(x) \approx 0.175$

Option C: $x = -0.3 \rightarrow g(x) \approx -0.001$ (Eliminate this option)

Therefore, Option A must be correct.

NESA Number:

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2024

Penrith High School
HSC TRIAL
EXAMINATION

Teacher: _____

Mathematics Advanced Section II Answer Booklet 1

Section II

90 marks

Attempt Questions 11–32

Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 17 (22 marks)

Booklet 2 – Attempt Questions 18 – 23 (22 marks)

Booklet 3 – Attempt Questions 24 – 29 (24 marks)

Booklet 4 – Attempt Questions 30 – 32 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

Booklet	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
1	/10 11, 12,13,14		/3 16		/9 15, 17	

Please turn over

Question 11 (2 marks)

Solve $\frac{3x+2}{5} - \frac{1-2x}{3} = 8$

2

$$\frac{3(3x+2) - 5(1-2x)}{15} = 8$$

$$9x + 6 - 5 + 10x = 120$$

$$19x = 119$$

$$x = \frac{119}{19}$$

NOTE: Very well done.

Only a few students made mistakes with the negatives.

Question 12 (2 marks)

Given that $\frac{12 + \sqrt{32}}{3\sqrt{2}} = a + b\sqrt{2}$. Show that $a = \frac{4}{3}$ and $b = 2$

2

$$\frac{12 + 4\sqrt{2}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{12\sqrt{2} + 8}{6}$$

$$= 2\sqrt{2} + \frac{4}{3}$$

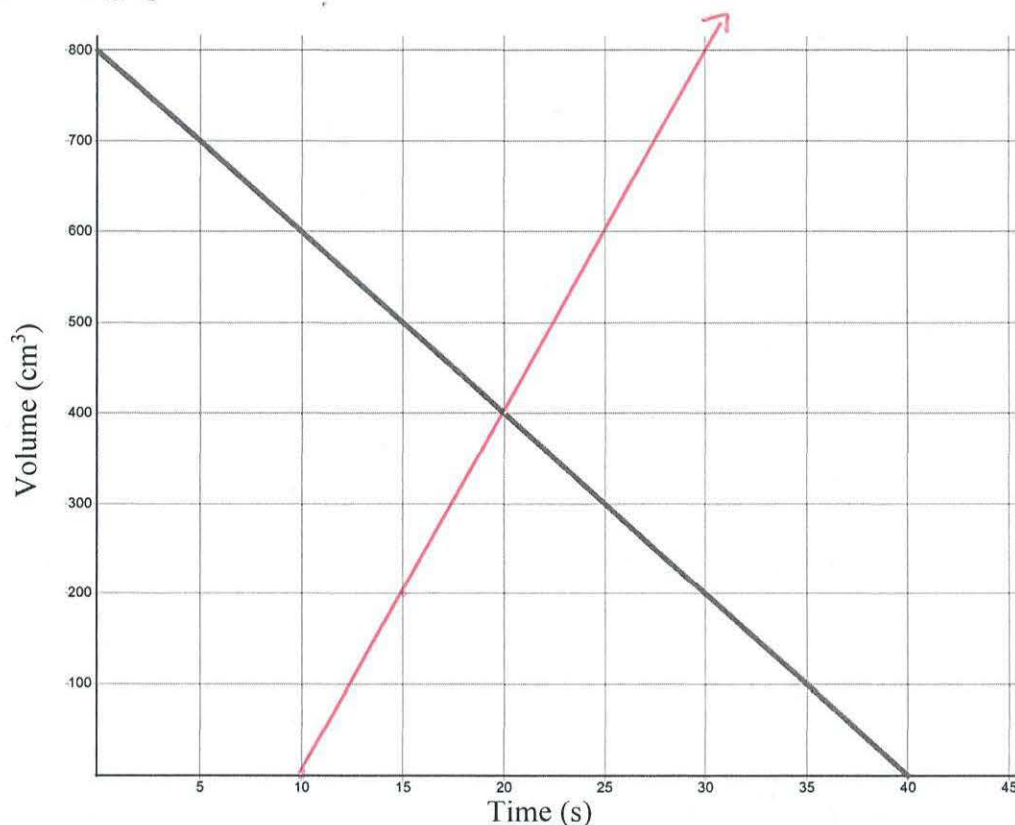
$$\therefore a = \frac{4}{3}, b = 2$$

NOTE: This is a SHOW question, so you must show clear working. i.e. the $\times \frac{\sqrt{2}}{\sqrt{2}}$ or equivalent must be shown.

Question 13 (3 marks)

Owais has two balloons, Balloon A and Balloon B. Initially Balloon A is filled with a volume, V , of 800 cm^3 of air and Balloon B is empty.

Owais accidentally puts a hole in Balloon A, which causes it to deflate over t seconds, as shown in the graph below.



- (a) At what rate is Balloon A deflating?

1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{800 - 700}{0 - 5} = -20 \text{ cm}^3/\text{s}$$

- (b) After 10 seconds of Balloon A deflating, Owais' mum starts to inflate Balloon B. She does this at a rate of $40 \text{ cm}^3/\text{s}$.

2

By drawing a line on the grid (above), find the value of t when the two balloons contain the same amount of air.

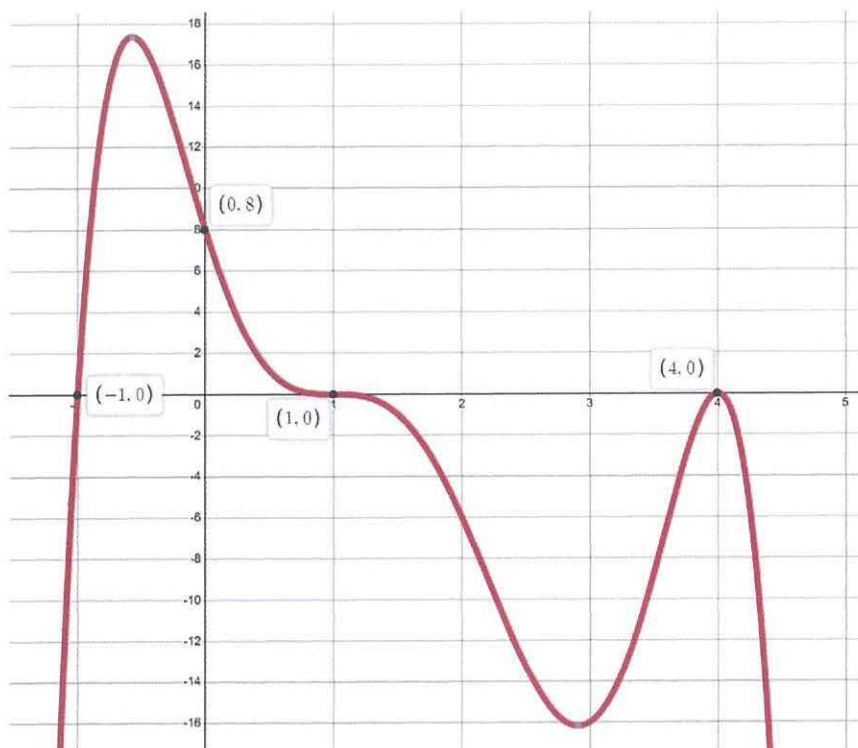
$$40 \text{ cm}^3/\text{s} = 200 \text{ cm}^3 \text{ in } 5 \text{ seconds}$$

$$t = 20 \text{ seconds}$$

Question 14 (3 marks)

Find the equation of the polynomial with degree 6.

3



$$y = a(x+1)(x-1)^3(x-4)^2$$

2 marks for
identifying roots and
respective powers.

Substitute (0, 8)

$$8 = a(0+1)(0-1)^3(0-4)^2$$

$$8 = a(1)(-1)^3(-4)^2$$

$$8 = a(-16)$$

$$\therefore a = \frac{-8}{16}$$

1 mark for
calculating 'a'

$$= -\frac{1}{2}$$

$$y = -\frac{1}{2}(x+1)(x-1)^3(x-4)^2$$

Question 15 (4 marks)

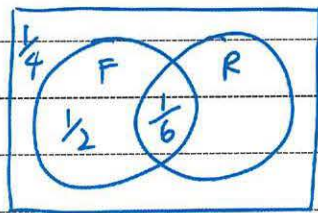
The probability that Darcy wants to play with a football outside on any particular day is $P(F)$.
The probability that it rains on any particular day is $P(R)$.

It is given that $P(F) = \frac{2}{3}$, $P(F \cap R') = \frac{1}{2}$ and $P(F' \cap R') = \frac{1}{4}$.

- (a) Is Darcy wanting to play with a football outside independent from whether or not it is raining? Justify your answer.

2

$$P(F) = \frac{2}{3}, P(F') = \frac{1}{3}, P(F \cap R') = \frac{1}{2}, P(F' \cap R') = \frac{1}{4}$$



$$\begin{aligned} P(F \cap R) &= P(F) - P(F \cap R') \\ &= \frac{2}{3} - \frac{1}{2} \\ &= \frac{1}{6} \end{aligned}$$

1 mark calculating additional probabilities

$$\begin{aligned} P(R) &= 1 - (1/4 + 1/2) \\ &= 1/4 \end{aligned}$$

$$P(F \cap R) = P(F) \times P(R) \quad \text{or} \quad P(F \cup R) = P(F) + P(R) - P(F \cap R)$$

$$\text{LHS} = 1/6$$

$$\text{LHS} = 3/4$$

1 mark

$$\text{RHS} = 2/3 \times 1/4$$

$$\text{RHS} = 2/3 + 1/4 - 1/6$$

justify events are independent.

$$= 1/6 = \text{LHS}$$

$$= 3/4 = \text{LHS}$$

$\therefore$ Darcy wanting to play football outside is independent of the weather.

- (b) Given that these probabilities stay the same every day, how many days would it take to ensure a 98% chance of Darcy wanting to play football at least once?

2

$$1 - P(F')^n = 0.98$$

$$1 - \left(\frac{1}{3}\right)^n = 0.98$$

1 mark forming equation required

$$\left(\frac{1}{3}\right)^n = 0.02$$

$$\log_{1/3} 0.02 = n$$

1 mark for using log laws

$$\therefore n = \frac{\log 0.02}{\log (1/3)}$$

$$= 3.56$$

$$\div 4 \text{ days}$$

Question 16 (3 marks)

- (a) Find $\frac{d}{dx}(x^3 + 1)^4$.

2

$$\begin{aligned}\frac{d}{dx}(x^3 + 1)^4 &= 4(x^3 + 1)^3 \times (3x^2) \\ &= 12x^2(x^3 + 1)^3\end{aligned}$$

* This question was done well.

- (b) Hence, find $\int 4x^2(x^3 + 1)^3 dx$.

1

$$\int 4x^2(x^3 + 1)^3 dx =$$

$$\frac{1}{3} \int 12x^2(x^3 + 1)^3 dx$$

$$= \frac{1}{3} (x^3 + 1)^4 + C$$

Some students had the incorrect value.

Some students forgot the '+C'.

Question 17 (5 marks)

On any given night, the number X of times that Isaac wakes up is a random variable with probability distribution given by:

x	0	1	2	3	4
$P(X = x)$	0.05	0.3	0.42	0.2	0.03

- (a) Show that Isaac's parents should expect him to wake up 1.86 times a night.

1

$$E(x) = (0 \times 0.05) + (1 \times 0.3) + (2 \times 0.42) + (3 \times 0.2) + (4 \times 0.03)$$

$$= 1.86$$

Markers' comments: This question was done well.

- (b) Calculate the standard deviation, correct to one decimal place.

Method 1: $\text{Var}(X) = E(X^2) - [E(X)]^2$ or

most students forgot this part.

$$\text{Var}(x) = 0^2 \times 0.05 + 1^2 \times 0.3 + 2^2 \times 0.42 + 3^2 \times 0.2 + 4^2 \times 0.03 - \mu^2$$

$$= 4.26 - 1.86^2$$

$$= 0.8004$$

$$\sigma = \sqrt{0.8004} = 0.89465... = 0.9 \text{ (to 1 dp)}$$

Method 2:

$x - \mu$	$0 - 1.86$	$1 - 1.86$	$2 - 1.86$	$3 - 1.86$	$4 - 1.86$
$(x - \mu)^2$	3.4596	0.7396	0.0196	1.2996	4.5796
$P(x)$	0.05	0.3	0.42	0.2	0.03

$$\text{Var}(x) = (3.4596 \times 0.05) + (0.7396 \times 0.3) + (0.0196 \times 0.42) + (1.2996 \times 0.2) + (4.5796 \times 0.03)$$

- (c) Find the probability that Isaac wakes up greater than the expected number of times for three days in a row.

$$\sigma = \sqrt{0.8004} = 0.9 \text{ (to 1 dp)}$$

$$P(x > 1.86) = P(x \geq 2)$$

$$= 0.42 + 0.2 + 0.03$$

$$= 0.65$$

* Some students were not sure what to do.

$$P(3 \text{ days in a row}) = 0.65^3$$

* This question was not done well.

2024**Penrith High School
HSC TRIAL
EXAMINATION**

NESA Number:

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Teacher: _____

Mathematics Advanced Section II Answer Booklet 2

Booklet 2 – Attempt Questions 18 – 23 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
 - Write your NESA ID above
-

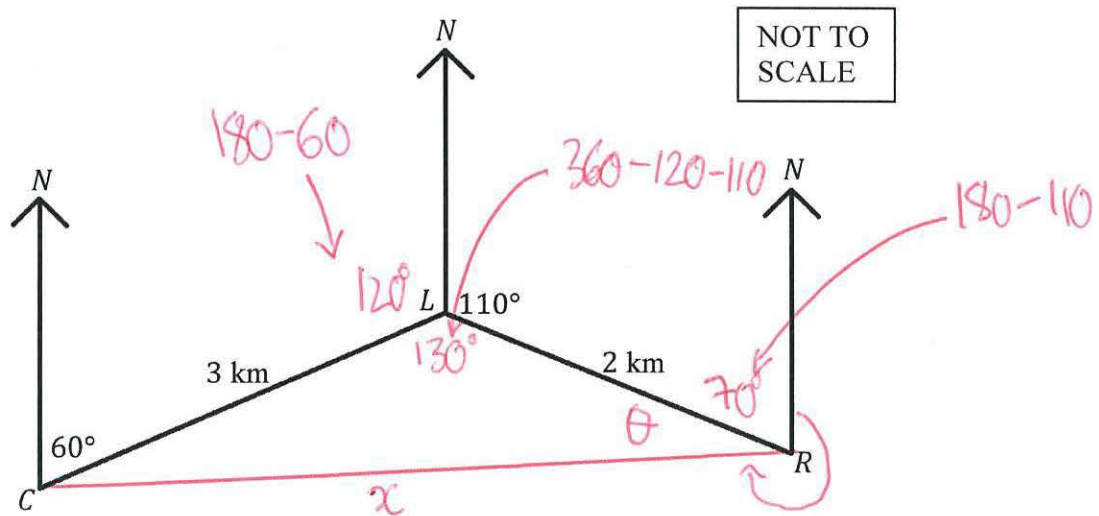
Booklet 2	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
	21 /3	18 /4	20. 23 /5		19, 22 /10	

Please turn over

Question 18 (4 marks)

Henry is going on a bushwalk. He walks 3 km on a bearing of $060^\circ T$ from his car, C, where he reaches a lookout, L. He then walks 2 km on a bearing of 110° to a rockpool, R.

4



Find the shortest distance that Henry needs to walk to return to his car and the bearing that he would be walking on from the Rockpool (to the nearest degree).

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$x^2 = 3^2 + 2^2 - 2 \times 3 \times 2 \times \cos 130^\circ$$

$$= 20.71$$

$$x = 4.55 \text{ km } (x > 0)$$

$$\frac{\sin \theta}{3} = \frac{\sin 130^\circ}{x}$$

$$\theta = \sin^{-1} \left(\frac{3 \sin 130^\circ}{x} \right)$$

$$= 30^\circ 19' 41.09''$$

$$\therefore 360 - 70 - \theta$$

$$= 260^\circ$$

$$\text{OR } S80^\circ W$$

NOTE: This was well done.

Some students made the error of assuming CR is a horizontal line i.e. $\angle NRC = 270^\circ$

Henry must walk 4.55 km on a bearing of 260°

Question 19 (4 marks)

Elena buys a car for \$25 000 and repays it over 4 years through equal monthly instalments. She is charged interest at 7.8% p.a. compounded monthly on the reducing balance loan.

Using the table of present value interest factors below, where r represents the monthly interest and N represents the number of repayments

Table of Present Value Interest Factors						
r	0.0060	0.0065	0.0070	0.0075	0.0080	0.0085
N						
45	39.33406	38.90738	38.48712	38.07318	37.66545	37.26383
46	40.09350	39.64965	39.21263	38.78231	38.35859	37.94133
47	40.84841	40.38714	39.93310	39.48617	39.04622	38.61311
48	41.59882	41.11986	40.64856	40.18478	39.72839	39.27924
49	42.34475	41.84785	41.35905	40.87820	40.40515	39.93975
50	43.08623	42.57113	42.06459	41.56645	41.07653	40.59470

- (a) Calculate the monthly repayment that Elena must pay to complete the loan after 4 years. 2

$$0.078 \div 12 = 0.0065$$

$$4 \times 12 = 48$$

$$\text{Present Value} = \text{Annuity} \times \text{Interest Factor}$$

$$25000 = a \times 41.11986$$

$$a = \$607.98$$

- (b) Calculate the total interest paid over the life of the loan. 2

$$607.98 \times 48 = 29183.04$$

$$29183.04 - 25000 = \$4183.04$$

$$\text{OR } \$4182.98$$

NOTE: This was very poorly done. A simple question if you remember the formula:

$$\text{Present/Future Value} = \text{Annuity} \times \text{Interest Factor (Number from table)}$$

Question 20 (3 marks)

Find the equation of the tangent to $y = \frac{x^4}{x-6}$ at the point $(2, -4)$.

3

$$y' = \frac{4x^3(x-6) - x^4}{(x-6)^2}$$

$$= \frac{3x^4 - 24x^3}{(x-6)^2}$$

at $x=2$

$$y' = \frac{3(2)^4 - 24(2)^3}{(2-6)^2}$$

$$= -9$$

$$\begin{aligned} u &= x^4 \\ u' &= 4x^3 \\ v &= x-6 \\ v' &= 1 \end{aligned}$$

NOTE: This was well done.

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -9(x - 2)$$

$$y + 4 = -9x + 18$$

$$y = -9x + 14$$

$$y + 9x - 14 = 0$$

Question 21 (3 marks)

For which values of k does $y = 2x^2 + kx + 8$ have real roots?

3

$$\Delta \geq 0$$

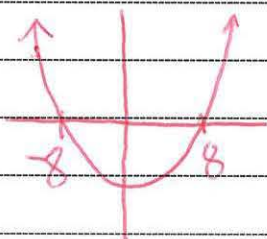
$$b^2 - 4ac \geq 0$$

$$k^2 - 4(2)(8) \geq 0$$

$$k \leq -8, k \geq 8$$

$$k^2 - 64 \geq 0$$

NOTE: This was poorly done.



$$\begin{aligned} x\text{-int} \\ k^2 - 64 &= 0 \\ k^2 &= 64 \\ k &= \pm 8 \end{aligned}$$

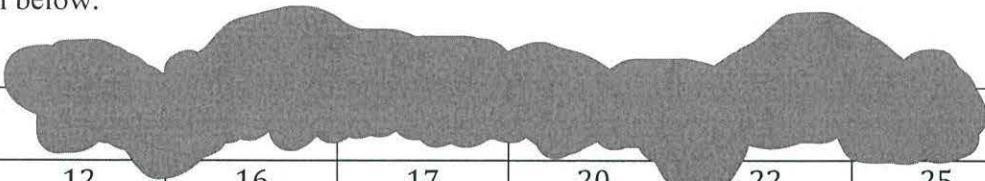
To solve a non-linear inequality you must draw a graph. In Advanced Maths this will only ever be a quadratic/parabola.

Question 22 (6 marks)

Aimee was baking some biscuits. She decided to make different sized biscuits, B , in grams and compare the time taken to bake them, t , in minutes. Aimee recorded her results in a table of values.

She then calculated the least squares regression line to be $t = 1.23B - 7.13$ and Pearson's correlation coefficient to be $r = 0.992$.

However, after doing these calculations she accidentally spilled her coffee on her notes and now can not see the row where she recorded the biscuit sizes. Conveniently she did not spill any coffee on the row where she recorded the time taken to cook. Her table of values (with coffee stain) is shown below.

Biscuit Size (B)						
Time to cook (t)	12	16	17	20	22	25

Aimee knows that in any bivariate dataset, the least squares regression line passes through the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the sample mean of the x -values and $\bar{y}$ is the sample mean of the y values.

- (a) Use this information to find the average weight of the biscuits.

2

$$\bar{t} = (12 + 16 + 17 + 20 + 22 + 25) \div 6$$

$$= 56 \div 6$$

NOTE: This was well done.

$$56 \div 6 = 1.23B - 7.13$$

$$\frac{7734}{300} = 1.23B$$

$$B = 20.97 \text{ grams (2 dp)}$$

- (b) Describe and interpret the gradient of the least squares regression line with reference to the context given.

1

The gradient of 1.23 indicates a cooking rate of 1.23 minutes per gram of biscuit.

NOTE: This was poorly done.

Question 22 continues on the next page.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{y\text{-values}}{x\text{-values}}$$

$$= \text{rate of change}$$

$$= y \text{ per } x.$$

- (c) Aimee uses the least squares regression line to estimate the time that it will take a 50 gram biscuit to cook. Discuss her use of the least squares regression line to estimate this value.

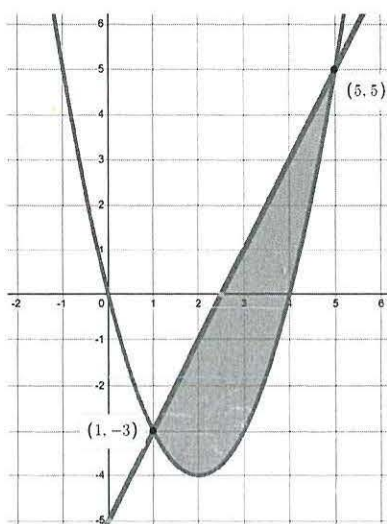
2

The cooking time and biscuit size have a strong correlation ($r = 0.992$), meaning a prediction/estimate should be accurate. However, this estimate is an extrapolation far beyond the dataset so Aimee can't be sure of its accuracy as any trend may not continue beyond the dataset.

Question 23 (2 marks)

The parabola $y = x^2 - 4x$ meets the line $y = 2x - 5$ at the points $(1, -3)$ and $(5, 5)$ as shown in the diagram.

2



NOTE: Discuss means provide points for and/or against. This question is worth 2 marks. You are able to and should provide a reason for (high r-value) and against (extrapolation well beyond the gathered data)

Find the area enclosed by the parabola and the line.

$$\int_1^5 (2x - 5) - (x^2 - 4x) dx$$

$$= \int_1^5 -x^2 + 6x - 5 dx$$

$$= \left[-\frac{x^3}{3} + \frac{6x^2}{2} - 5x \right]_1^5$$

$$= \left(-\frac{(5)^3}{3} + 3(5)^2 - 5(5) \right) - \left(-\frac{(1)^3}{3} + 3(1)^2 - 5(1) \right)$$

$$= \frac{32}{3} \text{ units}^2$$

NOTE: This was well done.

NESA Number:

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2024

**Penrith High School
HSC TRIAL
EXAMINATION**

Teacher: _____

Mathematics Advanced Section II Answer Booklet 3

Booklet 3 – Attempt Questions 24 – 29 (25 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

Booklet 3	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
	/13 24, 28, 29	/7 27		/5 25, 26		

Please turn over

Question 24 (5 marks)

The graph of the function $f(x) = x^3$ is translated up 9 units, left 4 units, then dilated vertically by a scale factor of $\frac{1}{3}$, to make a new graph, $g(x)$.

- (a) Find the equation, $g(x)$, of the new graph.

3

$$\begin{aligned} f(x) &= x^3 \\ f(x) + 9 \\ f(x+4) + 9 \\ \frac{1}{3}[f(x+4) + 9] \end{aligned}$$

1 mark for each transformation

$$\begin{aligned} g(x) &= \frac{1}{3}((x+4)^3 + 9) \\ &= \frac{1}{3}(x+4)^3 + 3 \end{aligned}$$

- (b) A point $P(a, b)$ on $f(x)$ is transformed to the point $Q(-1, 12)$ on $g(x)$. Find the values of a and b .

2

Use inverse operations

$$\begin{aligned} (-1, 12) & \downarrow \times 3 \quad (\text{inverse of dilate vertically } \frac{1}{3}) \\ (-1, 36) & +4 \downarrow \quad (\text{inverse of translate left 4 units}) \\ (3, 36) & \downarrow -9 \quad (\text{inverse of translate up 9 units}) \\ (3, 27) & \end{aligned}$$

1 mark for each value of a and b showing working

Question 25 (2 marks)

The difference in sound levels in decibels (dB) can be modelled by the equation

2

$$D_2 - D_1 = 10 \log_{10} \left(\frac{I_2}{I_1} \right)$$

Where D_1 and D_2 are the sound levels and I_1 and I_2 are their respective sound intensities measured in watts/m^2 .

A conversation has a sound level of 60 dB. A vacuum cleaner has an intensity that is 100 times more than the conversation. Calculate the sound level of the vacuum cleaner.

$$D_2 - 60 = 10 \log_{10} \left(\frac{100 I_1}{I_1} \right)$$

1 mark forming equation and simplifying

$$D_2 = 10 \log_{10} (100) + 60$$

$$= 10(2) + 60$$

$$\therefore D_2 = 80 \text{ dB}$$

1 mark correct use of log law and final solution.

Question 26 (3 marks)

Given $2 \log_4 y - \log_4 8 + \log_4 x = 2$, show that $y = \frac{8\sqrt{2}}{\sqrt{x}}$.

3

$$2 \log_4 y - \log_4 8 + \log_4 x = 2$$

$$\log_4 \left(\frac{y^2 x}{8} \right) = 2$$

1 mark simplify logs

$$\frac{y^2 x}{8} = 4^2$$

$$y^2 x = 128$$

$$y^2 = \frac{128}{x}$$

1 mark working towards y as the subject.

$$y = \pm \frac{\sqrt{128}}{\sqrt{x}}$$

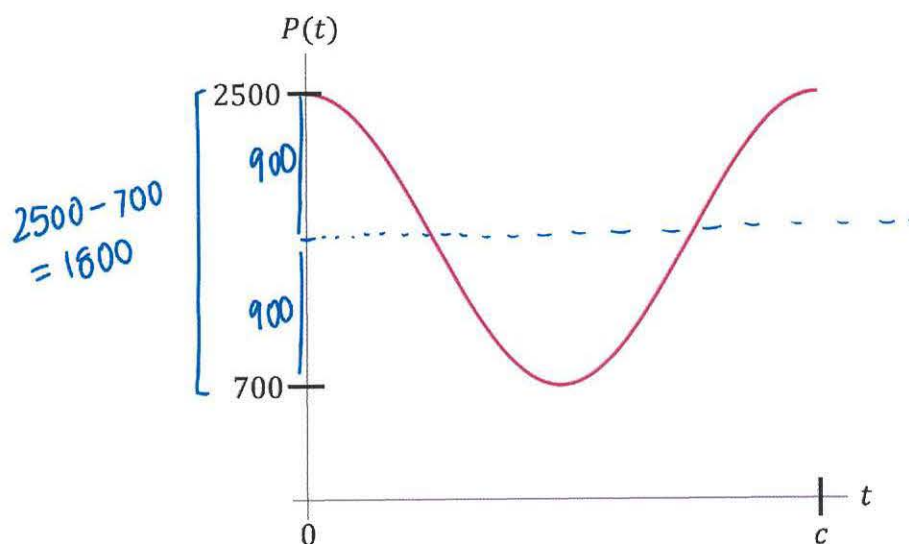
$$= \pm \frac{8\sqrt{2}}{\sqrt{x}} \text{ since } y > 0$$

1 mark writing y > 0 and final solution

$$y = \frac{8\sqrt{2}}{\sqrt{x}}$$

Question 27 (6 marks)

Anubhav is doing some research on the number of patients emergency waiting rooms across all NSW hospitals at any given time. He notices that the number of patients, P , can be modelled by the function $M(t) = a \cos\left(\frac{\pi t}{6}\right) + b$, where t is the time since 12 am. The maximum number of patients is 2500 and the minimum is 700. The graph of $P(t)$ is shown below.



- (a) What are the values of a and b in the equation and c on the graph?

3

$$a = 900 \text{ (amplitude)}$$

1 mark for

$$b = 900 + 700 = 1600 \text{ (centre)}$$

each variable

$$c = \frac{2\pi}{(\pi/6)} = 12$$

- (b) Between what times (to the nearest minute) each day are there less than 1000 patients in emergency waiting rooms across all NSW hospitals?

4

$$1000 = 900 \cos\left(\frac{\pi t}{6}\right) + 1600$$

$$\frac{\pi t}{6} = \pi + 0.841, \pi - 0.841$$

$$-600 = 900 \cos\left(\frac{\pi t}{6}\right)$$

$$t = \frac{6(\pi + 0.841)}{\pi}, \frac{6(\pi - 0.841)}{\pi}$$

$$\frac{-2}{3} = \cos\left(\frac{\pi t}{6}\right) \quad \text{1 mark to reach this step.}$$

$$t = 7.606 \dots, 4.39 \dots \quad \text{1 mark for solutions for } t.$$

$$\text{Reference } \angle = \cos^{-1}\left(\frac{2}{3}\right) \quad \begin{array}{c|c} \sqrt{5} & A \\ \sqrt{7} & C \end{array}$$

1 mark correct reference angle.

$$= 0.841 \dots$$

$\therefore$ There are less than 1000 patients between 4:24am to 7:36am and 4:24pm to 7:36pm. 1 mark for all times

Question 28 (5 marks)

Carlisle is playing a game, in which he has to position a bomb anywhere on a number plane. When the bomb explodes it will kill anything within its circular blast zone.

- (a) Find the position that Carlisle placed the bomb on the number plane and the radius of its circular blast zone, given its' blast zone has an equation $x^2 - 20x + y^2 + 10y = -76$.

3

$$x^2 - 20x + 100 + y^2 + 10y + 25 = -76 + 100 + 25$$

$$(x-10)^2 + (y+5)^2 = 49$$

$$(x-10)^2 + (y+5)^2 = 7^2$$

1 mark for
completing the square

$\therefore (10, -5)$ is the centre of
the blast zone, and the
blast zone has radius 7 units.

1 mark for
centre

1 mark for
radius

- (b) Determine if an enemy located at the coordinates $(2, -4)$ will be killed when the bomb explodes. Justify your decision with a diagram and/or calculations.

2

Calculate distance
between $(2, -4)$ and $(10, -5)$

$$d = \sqrt{(10-2)^2 + (-5-(-4))^2}$$

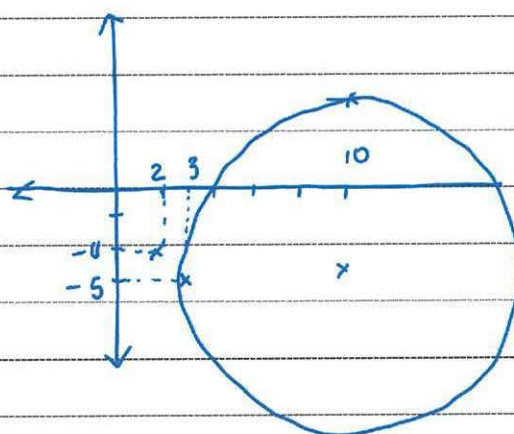
$$= \sqrt{8^2 + (-1)^2}$$

$$= \sqrt{65}$$

$$= 8.06...$$

The enemy is outside
of the blast zone and
will survive.

Diagram



2 marks for drawing diagram
or using the distance
formula.

Question 29 (3 marks)

Let $f(x) = \sqrt{3x - 15}$ and $g(x) = -6x - 4$.

- (a) Find $h(x)$, where $h(x) = f(g(x))$.

1

$$h(x) = \sqrt{3(-6x-4)-15}$$

$$= \sqrt{-18x-12-15}$$

1 mark for
correct substitution

$$h(x) = \sqrt{-18x-27}$$

- (b) State the domain for $h(x)$.

2

$$-18x - 27 \geq 0$$

$$-18x \geq 27$$

$$x \leq \frac{27}{-18}$$

$$-18$$

$$x \leq \frac{-3}{2}$$

$$2$$

1 mark for identifying
domain

1 mark for solving
inequality correctly.

$$(-\infty, \frac{-3}{2}]$$

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2024

Penrith High School
HSC TRIAL
EXAMINATION

Teacher: _____

Mathematics Advanced Section II Answer Booklet 4

Booklet 3 – Attempt Questions 30 – 32 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 4			32 /12			30, 31 /10

Please turn over

Question 30 (2 marks)

Jacinta is gifted \$5000 from her grandmother. The money is invested for 18 years, earning 7% p.a. in interest, compounded monthly. Calculate the effective annual interest rate of the investment over the 18 years. Answer as a percentage correct to two decimal places.

2

Method 1:

$$A = 5000 \left(1 + \frac{0.07}{12}\right)^{12 \times 18}$$

$$= 17562.6966$$

$$17562.6966 = 5000(1+r)^{18}$$

$$(1+r)^{18} = 3.512539...$$

$$1+r = \sqrt[18]{3.512539}$$

$$r = 1.072290081 - 1$$

$$= 0.072290$$

Method 2:

$$EAR = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{0.07}{12}\right)^{12} - 1$$

$$= 0.07229...$$

$$= 7.229\%$$

$$= 7.23\% \text{ (to 2dp)}$$

$r = 7.23\%$ (to 2dp) This question was not done well.

Question 31 (8 marks)

Ashley borrows \$500 000 from a bank in a reducing balance loan for which she is charged an interest rate of 6% per annum, compounded monthly. At the end of each month Ashley is required to make a repayment of \$4000.

The amount in the account immediately after the n th repayment can be determined using the recurrence relation

$$A_n = A_{n-1}(1.005) - 4000$$

where $n = 1, 2, 3 \dots$ and $A_0 = 500\,000$

- (a) Use the recurrence relation to determine the amount of money owing immediately after the third repayment.

2

$$A_0 = 500\,000$$

$$A_1 = A_0(1.005) - 4000$$

$$= 500\,000(1.005) - 4000$$

$$= 498\,500$$

$$A_2 = A_1(1.005) - 4000$$

$$= 498\,500(1.005) - 4000$$

$$= 496\,992.50$$

$$A_3 = A_2(1.005) - 4000$$

$$= 496\,992.50(1.005) - 4000$$

$$= 495\,477.46$$

∴ The amount owing after the third repayment is \$495 477.46

* Students did not use the previous values.

Question 31 continues on the next page.

(b) Show that $A_n = 500\,000(1.005)^n - 4000 \frac{(1.005^n - 1)}{0.005}$.

$$A_0 = 500\,000$$

$$A_1 = A_0(1.005) - 4000$$

$$= 500\,000(1.005) - 4000$$

$$A_2 = A_1(1.005) - 4000$$

$$= [500\,000(1.005) - 4000]1.005 - 4000$$

$$= 500\,000(1.005)^2 - 4000(1.005) - 4000$$

$$A_3 = A_2(1.005) - 4000$$

$$= [500\,000(1.005)^2 - 4000(1.005) - 4000]1.005 - 4000$$

$$= 500\,000(1.005)^3 - 4000(1.005)^2 - 4000(1.005) - 4000$$

$$A_n = 500\,000(1.005)^n - 4000(1.005)^{n-1} - 4000(1.005)^{n-2} - \dots - 4000$$

$$= 500\,000(1.005)^n - 4000(1.005^{n-1} + 1.005^{n-2} + \dots + 1.005 + 1)$$

$$= 500\,000(1.005)^n - 4000(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-2} + 1.005^{n-1})$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

where $a=1$, $r=1.005$,
n terms.

$$\therefore A_n = 500\,000(1.005)^n - 4000 \left(\frac{1(1.005^n - 1)}{1.005 - 1} \right)$$

$$= 500\,000(1.005)^n - 4000 \left(\frac{1.005^n - 1}{0.005} \right)$$

* Some students did not show the A_n with the GP

* Some students did not show S_n .

(c) Hence, find how many months it will take for Ashley to repay the \$500 000.

3

$A_n = 0$ for loan to be paid.

$$A_n = 500\,000(1.005)^n - 4000 \left(\frac{1.005^n - 1}{0.005} \right)$$

$$0 = 500\,000(1.005)^n - 4000 \left(\frac{1.005^n - 1}{0.005} \right)$$

Solve for n ,

$$0 = 500\,000(1.005)^n - 800\,000(1.005^n - 1)$$

$$500\,000(1.005)^n = 800\,000(1.005^n - 1)$$

$$(1.005)^n = 1.6(1.005^n - 1)$$

* Algebraic errors

$$1.005^n = 1.6 \times 1.005^n - 1.6$$

$$1.6 = 1.6 \times 1.005^n - 1.005^n$$

$$1.6 = 1.005^n(1.6 - 1)$$

$$1.005^n = \frac{8}{3}$$

$$n = \log_{1.005} \left(\frac{8}{3} \right)$$

$$\text{Using change of base} = \frac{\log\left(\frac{8}{3}\right)}{\log 1.005} = 196.65585 = 197 \text{ months.}$$

Question 32 (13 marks)

Let $f(x) = 3x^3 e^x$.

$$f'(x) = vu' + uv'$$

(a) Show that $f'(x) = 3e^x(3x^2 + x^3)$.

2

$$f'(x) = e^x(9x^2) + 3x^3(e^x)$$

$$= 9x^2e^x + 3x^3e^x$$

$$= 3e^x(3x^2 + x^3)$$

$$\begin{array}{l|l} u = 3x^3 & v = e^x \\ u' = 9x^2 & v' = e^x \end{array}$$

* This question was done well.

Question 32 continues on the next page.

(b) Find any points of inflection of $f(x)$.

(You may state the values of the points correct to 1 decimal place.)

$$f(x) = 3x^3 e^x$$

$$f'(x) = 3e^x (3x^2 + x^3)$$

$$f''(x) = 3e^x (6x + 3x^2) + (3x^2 + x^3) 3e^x$$

$$= 18e^x x + 9e^x x^2 + 9e^x x^2 + 3x^3 e^x$$

$$= 18e^x x + 18x^2 e^x + 3x^3 e^x$$

$$= 3xe^x (6 + 6x + x^2)$$

$f''(x) = 0$ for possible points of inflection.

$$3e^x x (x^2 + 6x + 6) = 0$$

Using Null factor law,

$$3e^x x (x^2 + 6x + 6) = 0$$

$$3e^x \neq 0$$

$$x = 0$$

or

$$x^2 + 6x + 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

no solution

$$\sin e^x > 0$$

$$x = \frac{-6 \pm \sqrt{36 - 4(1)6}}{2}$$

Some students forgot this

$$f(0) = 3(0)^3 e^0$$

$$= 0$$

$$= \frac{-6 \pm \sqrt{12}}{2}$$

$$f(-1.3) = -1.7 \text{ (to 1 dp)}$$

$$x = -3 \pm \sqrt{3}$$

$$f(-4.7) = -2.83$$

$$x = -1.26, x = -4.7$$

$$= -2.8 \text{ (to 1 dp)}$$

$$x = -1.3$$

$\therefore$ Possible points of inflection are:

$(0, 0)$, $(-1.3, -1.7)$ and $(-4.7, -2.8)$

Test concavity changes.

x	-5	-4.7	-3	-2	-1.26	-1	0	1
$f''(x)$	-0.1	0	1.3	1.6	0	-0.04	0	106.0
	<0		>0	>0		<0		>0

← Input values

Do not use gradients!

$\therefore$ concavity change around $x = 0, -1.3, -4.7$.

$\therefore$ Points of inflection $(0, 0)$, $(-1.3, -1.7)$ and $(-4.7, -2.8)$

Question 32 continues on the next page.

- (c) Show that the only stationary points of $f(x)$ are $(0, 0)$ and $(-3, -81e^{-3})$ and determine their nature. 4

Stationary points occur when $f'(x) = 0$

$$f'(x) = 3e^x(3x^2 + x^3)$$

$$3e^x(3x^2 + x^3) = 0$$

$$3e^x \neq 0$$

since $3e^x > 0$

no solution

* Students forgot this part

$$\text{or } 3x^2 + x^3 = 0$$

$$x^2(3+x) = 0$$

$$x^2 = 0 \text{ or } 3+x = 0$$

$$x = 0 \text{ or } x = -3$$

$$f(0) = 3(0)^3 e^0 = 0$$

$$f(-3) = 3(-3)^3 e^{-3} = -81e^{-3}$$

Students found these correctly.

Nature of stationary points.

$$f''(x) = 3e^x x(6 + 6x + x^2)$$

$$f''(-3) = 3e^{-3}(-3)(6 + (6)(-3) + (-3)^2) = 27e^{-3}$$

$$= 1.34 > 0 \text{ concave up}$$

$\therefore$ minimum turning point at $(-3, -81e^{-3})$

x	-4	-3	-2
$f'(x)$	-0.87	0	4.87
	\	-	/

From part (b), $(0, 0)$ is a point of inflection

x	-1	0	1
$f(x)$	2.21	0	32.61
	/	-	/

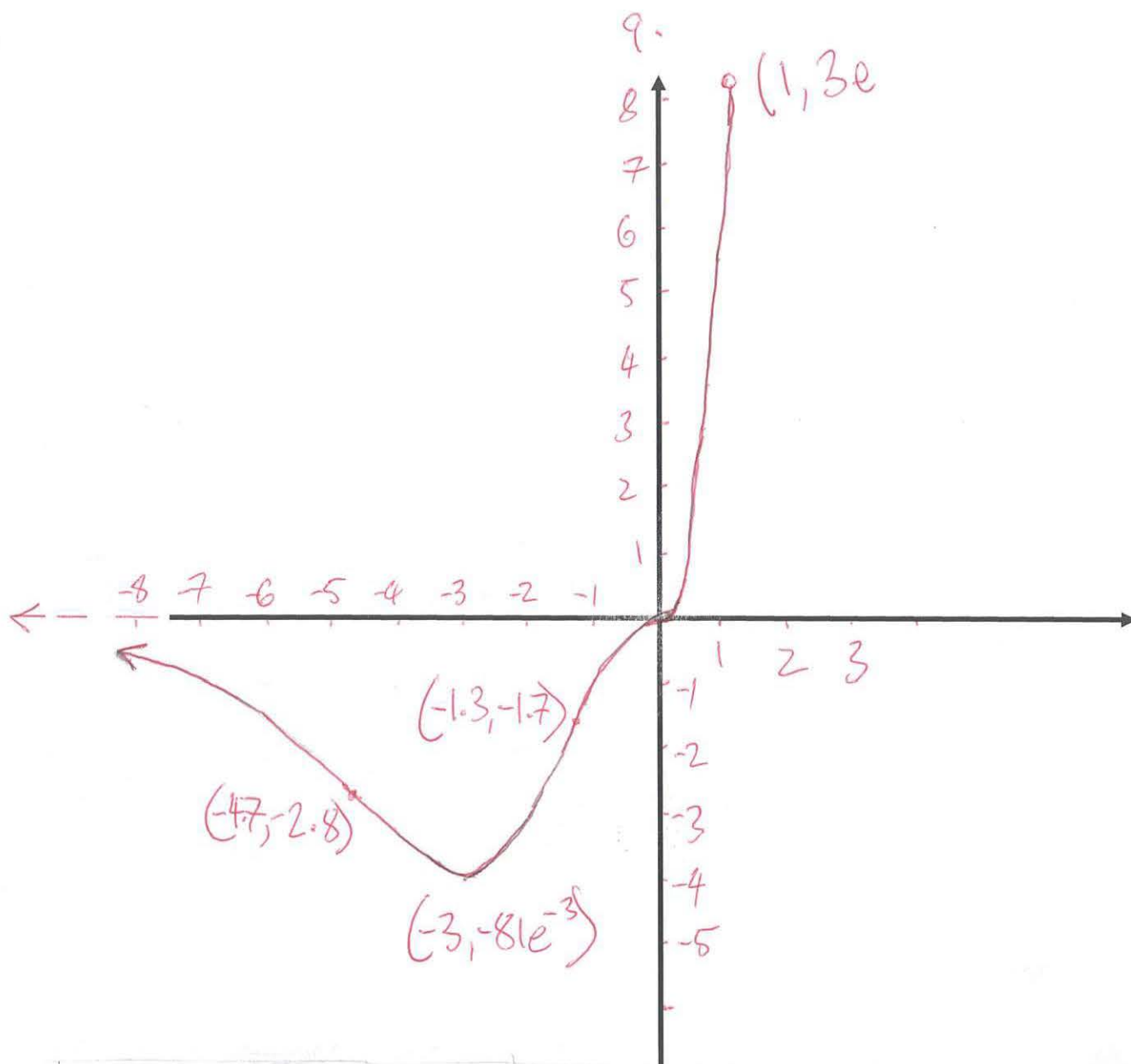
$\therefore$ Horizontal point of inflection

* Overall, this question was done well.

Question 32 continues on the next page.

- (d) Hence sketch the curve of $f(x) = 3x^3 e^x$ for $x < 1$, showing any stationary points, points of inflection and intercepts with the axes.

3



$$\begin{aligned}
 x=1 \\
 y &= 3(1)^3 e^1 \\
 &= 3e \\
 &\approx 8.15
 \end{aligned}$$

$$\begin{aligned}
 x\text{-int when } y &= 0 \\
 0 &= 3x^3 e^x \\
 e^x &\neq 0 \quad 3x^3 = 0 \\
 x &= 0
 \end{aligned}$$

y-int already found
 $(0,0)$

$$\begin{array}{c|c|c|c}
 \text{as } x \rightarrow -\infty & & & \\
 \hline
 x & -6 & -10 & -100 \\
 \hline
 f(x) & -1.6 & -0.14 & -1.1 \times 10^{-37} \\
 \hline
 \end{array}$$

Clearly $f(x) \rightarrow 0$ from below.